

Experimental Study of Gravity Measurement with a Video-Based Laboratory Pendulum with Tracker Software: Comparison of Weighted and Unweighted Tests

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Abstract—This research aims to determine the value of the Earth's gravitational acceleration (g) using the mathematical pendulum swing method with an unweighted linear regression approach, weighted linear regression, and the 4th-order Runge-Kutta numerical method. The data used are the results of measuring the swing period of the pendulum for various lengths of string. The analysis was carried out by calculating the average value of gravitational acceleration using multiple methods. The results show that weighted linear regression provides more consistent and accurate estimates than unweighted linear regression, with a high coefficient of determination (R^2) value. The Runge-Kutta numerical method is also used to predict swing periods with a more in-depth mathematical approach, producing values supporting experimental data trends. Overall, this research makes an average value of gravitational acceleration of around 9.11 m/s^2 , close to the expected theoretical value. These findings show that the mathematical pendulum swing method can be used effectively to measure the Earth's gravitational acceleration with sufficient accuracy and provide an essential contribution in the context of physics education regarding the application of basic principles in physics experiments.

Index Terms—gravity, Runge-Kutta, unweighted data, video tracker, weighted data.

I. INTRODUCTION

GRAVITY is one of the fundamental forces of nature that affect all objects with mass [1]. Understanding gravity is essential in a theoretical context and everyday practical applications, such as physics, engineering, astronomy, and geophysics [2]. The value of gravitational acceleration on the Earth's surface is an important parameter often needed in various scientific experiments and calculations [3]. One classic method for determining gravity values is using a simple pendulum, which utilizes the principle of simple harmonic oscillations [4]. In addition, understanding gravity also plays a vital role in space exploration, where the gravitational influence of various celestial bodies, such as planets and moons, must be considered in mission planning [5]. In geophysics, local variations in Earth's gravity can provide valuable information about subsurface structures, such as the presence of minerals or oil reservoirs [6]. Gravity also

influences Earth's water flow and pressure distribution, which impacts volcanic activity and earthquakes [7]. Gravity calculations are needed in civil engineering to design buildings and bridges to withstand loads and remain stable [8]. Overall, gravity is a fundamental and inseparable concept from many aspects of science and technology, making it a subject that continues to be studied for deeper understanding and broader application [9]. The pendulum is a simple but effective tool for measuring gravity [10]. The basic concept behind using a pendulum for measuring is the relationship between the swing period of the pendulum and the length of the pendulum string [11] [12]. Simple pendulum theory states that the swing period of a pendulum is directly proportional to the square root of the string length and inversely proportional to the square root of the acceleration due to gravity [13] [14]. From this relationship, we can extract the value of (g) by measuring the period (T) and the length (l) [15] [16] [17] [18].

However, in practice, measuring g using a pendulum is a complex task [19] [20]. Various factors, including accuracy in measuring pendulum length, swing amplitude, and environmental conditions such as air resistance, can significantly influence the accuracy of measurement results [21]. One factor that is often considered is the mass of the weight on the pendulum [22], [23]. Although ideal theory states that the mass of the weight does not affect the swing period, in reality, the presence of additional weight can influence other factors such as friction and air resistance [24]. Additionally, the ideal pendulum movement should be a small oscillatory movement; when the swing amplitude is too large, the linear approach used in period calculation begins to lose its accuracy due to the influence of non-linearity [25], [26]. Environmental conditions also play an important role; for example, temperature variations can affect the length of a pendulum string, resulting in small but significant changes in the swing period [27]. Air resistance, even if small, can cause damping of oscillations that change the overall system dynamics [28], [29]. Furthermore, geographic location can influence gravity measurements because the value of gravitational acceleration varies slightly on the Earth's surface due to factors such as altitude and uneven distribution of the Earth's mass [30]. To obtain more accurate results, experiments are often carried out in vacuum conditions to eliminate the influence of air resistance and with the use of highly sensitive sensors to precisely measure the swing length and period [31]. All

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these factors show that the concept of measuring gravity with a pendulum, while simple in theory, presents a complex challenge in practice that requires meticulous attention to many practical details to obtain accurate and reliable results [32].

This study aims to further explore the effects of taking unweighted data and weighted data on gravity measurements using a pendulum. By conducting experiments that compare the conditions of a weighted and unweighted pendulum, it is possible to evaluate and influence the measurement results. In addition, this research will examine whether additional weight can increase or reduce the accuracy of g measurements. This experiment will involve measuring the swing period of the pendulum under various different mass conditions, as well as analyzing the influence of environmental variables such as temperature and air resistance. The measurement results will be compared with theoretical values to determine the level of error that occurs. In addition, this research will also utilize modern technology, such as high-precision motion sensors and data analysis software, to improve measurement accuracy and minimize experimental errors. Statistical analysis will be used to evaluate the significance of the results obtained and determine whether there is a strong correlation between weight mass and changes in swing period. In this study, computer simulations will also be carried out to model the influence of the weight mass on the pendulum under various conditions, providing a more comprehensive view of the system dynamics. It is hoped that the findings from this research can make a significant contribution to gravity measurement methods and can be applied in the context of education and further scientific research. The final results are expected to provide practical recommendations for improving the accuracy of gravity measurements using pendulums, as well as provide new insights into mass effects in simple harmonic oscillation systems, potentially revolutionizing the way we measure gravity.

II. THEORETICAL BACKGROUND

Mathematical swing is a concept in physics that describes the oscillatory movement of a pendulum suspended from a thread [33]. These swings follow the basic principles of second-order differential equations, which are at the heart of much analysis in simple harmonic motion physics [34]. However, in reality, this mathematical swing motion is only partially harmonious [35]. It is caused by friction between the pendulum and the surrounding air, which causes the pendulum to slow down until it finally stops [36]. To minimize the effects of friction and encourage harmonic oscillations, several ideal conditions must be met, such as ensuring the weight of the thread is much smaller than the weight of the pendulum and keeping the angle of deviation of the pendulum small [37].

A more in-depth discussion of mathematical swings includes various aspects that influence the dynamics of pendulum movement [38]. First, we need to understand how the differential equations that govern the motion of a pendulum function in practice [39]. This equation shows that the motion of an ideal pendulum is simple harmonic

motion with a period that depends on the length of the string and the acceleration due to gravity [40]. However, due to air friction, the oscillation energy decreases over time, causing the pendulum's motion to stop, meaning the pendulum experiences damped oscillatory motion eventually [41]. Several steps can be taken to reduce the damping effect and approach ideal conditions, such as using a very light rope and keeping the pendulum deviation angle small [42]. Slight deviation angles are essential because, at small angles, the assumption that the deviation angle is proportional to the displacement remains valid, allowing the use of simple harmonic equations of motion [43].

Experiments with mathematical pendulums are often used to measure the Earth's gravitational acceleration [44], [45]. In this experiment, maintaining ideal conditions is essential to obtain accurate results [46]. Determining the Earth's gravitational acceleration using a mathematical pendulum involves measuring the length of the string and the period of oscillation, followed by data analysis to determine the relationship between the size of the string and the period [47], [48]. The data obtained is then analyzed to ensure that this relationship follows theory, and the value of gravitational acceleration can be calculated from the slope of the resulting line [49]. In addition to simple linear regression analysis, numerical methods such as Runge-Kutta can be used to estimate the value of gravitational acceleration [50]. This method is helpful in conditions where the linearity assumption is only partially valid [51]. In more sophisticated experiments, the damping effect can also be analyzed [52]. The damping coefficient can be determined by measuring how the oscillation amplitude decreases over time, which is essential for a deeper understanding of the dynamics of oscillatory motion and the factors that influence it [53], [54].

The importance of research on mathematical swings cannot be overstated [55]. In addition to providing a reliable method for measuring gravitational acceleration, this experiment is also an excellent illustration of the basic principles of physics [56]. By understanding the dynamics of simple movements, such as the swing of a pendulum, we can build a foundation for studying more complex systems in physics [57]. This research also has significant educational implications because pendulum experiments are often used in physics education to teach basic concepts such as simple harmonic motion, damping, and data analysis [58]. Through these experiments, students can learn how to collect and analyze data and apply theoretical concepts in real situations [59].

In a broader context, understanding the basic principles of physics illustrated by the swing of a pendulum is the first step toward understanding the dynamics of more complex systems, such as planets' motion and quartz's swing in clocks [60]. Therefore, research on mathematical swings is an academic exercise and an essential basis for further scientific exploration [61]. This research also provides insight into reducing damping effects in various mechanical systems [62]. In modern technology, from mechanical clocks to resonant devices in telecommunications, controlling and understanding

damping is critical to improving system efficiency and reliability [63].

Numerical approaches such as the Runge-Kutta method used in pendulum swing analysis show how computational techniques can be used to solve problems that cannot be solved analytically [64]. This approach is critical in today's digital era, where computing is essential to scientific and engineering research [65]. In conclusion, mathematical swings are handy tools in physics to help understand the basic principles of oscillatory motion [66]. Although, in practice, it is only partially harmonious due to friction, various analysis methods, such as linear regression and numerical methods, can be used to obtain accurate and reliable results [67]. This research enriches our understanding of the dynamics of motion and significantly contributes to physics education and the application of modern technology [68].

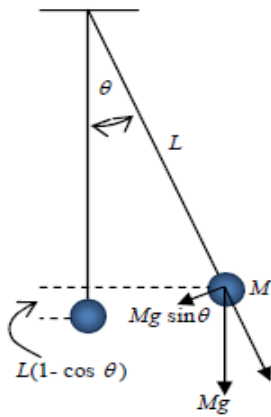


Fig 1. Mathematical Swing

Figure 1 shows that when the pendulum is at the deviation angle θ (or maximum deviation), the forces F_1 and F_2 work. F_1 direction is to the right, and F_2 direction is to the left. The mathematical formula that describes this relationship is presented in Equation 1.

$$F_1 = m \frac{d^2 S}{dt^2} \dots\dots\dots (1)$$

The length of the bow $s=l\theta$ with l the length of the string so that becomes Equation 2.

$$m \frac{d^2 S}{dt^2} = m \frac{d^2 l \theta}{dt^2} = ml \frac{d^2 \theta}{dt^2} \dots\dots\dots (2)$$

This force is balanced by the controlling force generated by the load deflecting at an angle θ , being Equation 3.

$$F_2 = -wsin \theta = -mg \sin \theta \dots\dots\dots (3)$$

The negative sign indicates that the direction of F_2 is opposite to that of F_1 . If the angle θ is small (less than 15°), then $\sin \theta \approx \theta$, and thus Equation 3 simplifies to Equation 4.

$$F_2 = -mg\theta \dots\dots\dots (4)$$

In a state of equilibrium, $F_1=F_2$ so that Equation 4 becomes Equation 5.

$$ml \frac{d^2 \theta}{dt^2} = -mg\theta \dots\dots\dots (5)$$

$$\frac{d^2 \theta}{dt^2} = -\frac{g}{l} \theta$$

The harmonic oscillation equation has a frequency. Considering the period T , the period follows the given Equation 6.

$$T^2 = \frac{4\pi^2}{g} l \dots\dots\dots (6)$$

A comprehensive set of data is obtained by varying the string lengths, measuring the corresponding periods for each string length, and repeating the experiment several times. From this data set, it is possible to form a linear regression equation that describes the relationship between string length and oscillation period. Suppose l denotes the size of the string and the period of oscillation by T . In that case, we can express the relationship as a linear regression, where l and T are interrelated variables. The Earth's gravitational acceleration can be calculated from the slope of the graph resulting from this linear regression, according to the equation that connects the length of the string, the period of oscillation, and the acceleration of gravity. The Earth's gravitational acceleration can be determined from the slope, as described in Equation 7.

$$g = \frac{4\pi^2}{a} \dots\dots\dots (7)$$

Meanwhile, the gravity correction follows Equation 8.

$$s_g = \frac{\partial g}{\partial a} s_a \dots\dots\dots (8)$$

III. METHOD

The methodology used in this research involved a series of experiments with varying rope lengths. Each experiment will be repeated several times to ensure data consistency and reliability. The measurement results will be analyzed statistically to determine significant differences between weighted and unweighted conditions. In addition, measurement uncertainty calculations will be performed to evaluate the accuracy of the results. In the data analysis process, the video tracking tool will more accurately track and analyze the movement of the pendulum. This technology allows detailed observation of the oscillation period and swing amplitude, thereby minimizing manual measurement errors and providing more accurate data.

A. Unweighted Linear Regression

This method uses linear regression to model the relationship between these two variables by minimizing the squared difference between the observed value and the value predicted by the model. The rope length and swing period data obtained from the experiment are then fitted into a linear equation, where the slope of the regression line and the intercept are calculated.

The first step in linear regression is to calculate the average of the string length and period. Then, the slope of the regression line is calculated to determine the extent to which

changes in rope length affect the swing period. Once the slope is obtained, the intercept is estimated to complete the linear regression line equation. The graph of the relationship between rope length and swing period will show a straight line resulting from the regression equation.

The coefficient of determination (R^2), a key metric, is used to measure the goodness of fit of the regression model to the existing data. A value close to 1 indicates a high level of accuracy in explaining data variability. In our weightless linear regression analysis of pendulum swing experiments, we often observe a strong relationship between string length and swing period. This relationship is instrumental in calculating the Earth's gravitational acceleration. The Earth's gravitational acceleration is derived from the slope of the regression line, and the results are then compared with theoretical values to evaluate the accuracy of the experiment.

B. Weighted Linear Regression

Unweighted linear regression is a statistical method used to model the relationship between two variables by minimizing the squared difference between the observed value and the value predicted by the model. In the context of a pendulum swing experiment, this method is applied to understand how the length of the string (L) affects the swing period (T). One can construct a linear model that describes this relationship by collecting data from various string lengths and measuring the corresponding swing periods.

The analysis begins by calculating the rope length and swing period average. Next, the regression line and intercept slope are calculated to obtain the linear line equation that best fits the existing data. The regression results graph shows a straight line that reflects the relationship between string length and swing period. The level of fit of the model to the data was assessed using the coefficient of determination (R^2). An R^2 value close to one indicates that the regression model can explain the variability in the data.

C. Runge Kutta 4th Order

To carry out numerical analysis using the Runge-Kutta method, it is necessary to understand the context and purpose of using it in a given situation. The Runge-Kutta method (Equation 9) is generally used to solve differential equations. However, the data provided is in the form of a pair (l_i, T_i), which describes the length of the pendulum (l) and the swing period (T). In this context, we must formulate the relevant differential equations for numerical analysis using the Runge-Kutta method.

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \dots\dots\dots (9)$$

With

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_i + h, y_i + k_3)$$

However, because the data provided is a discrete pair, it is necessary to model the relationship between the two, such as using a model that approaches exponential or polynomial form.

IV. RESULT

The results of data sampling using Video Tracker are presented in Fig. 2.

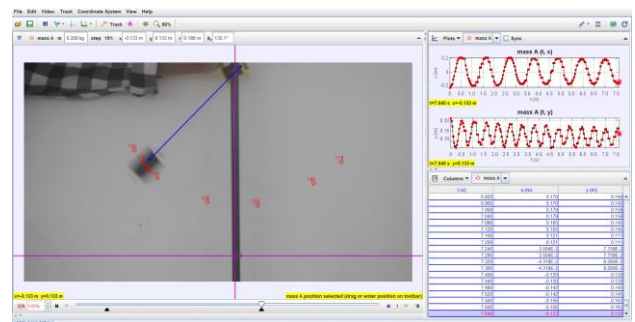


Fig 2. Video Tracker Snippet

Data from measurements of the oscillatory motion of the pendulum for one swing at five rope length variations are presented in Table 1.

TABLE I
DATA FROM MEASUREMENTS OF THE OSCILLATORY MOTION OF THE PENDULUM FOR ONE SWING OF FIVE-STRING LENGTH VARIATIONS

l_i (m)	T_i (s)
0.32	1.166655
0.39	1.233321
0.47	1.433319
0.54	1.533318
0.66	1.633317

Data from measurements of the oscillatory motion of the pendulum for five swings at five rope length variations are presented in Table II.

TABLE II
DATA FROM MEASUREMENTS OF THE OSCILLATORY MOTION OF THE PENDULUM FOR FIVE SWINGS AT FIVE VARIATIONS IN STRING LENGTH

l_i (m)	T_i (s)	l_i (m)	T_i (s)	l_i (m)	T_i (s)	l_i (m)	T_i (s)	l_i (m)	T_i (s)
0.66	1.633317	0.54	1.533318	0.47	1.433319	0.39	1.299987	0.32	1.233321
	1.699983		1.533318		1.399986		1.33332		1.166655
	1.699983		1.466652		1.433319		1.33332		1.199988
	1.66665		1.499985		1.433319		1.299987		1.166655
	1.699983		1.566651		1.399986		1.299987		1.166655

A. Unweighted Linear Regression

Mathematical pendulum swings are one of the methods used to determine the magnitude of the Earth's gravitational acceleration g in a place after knowing the magnitude of the period of the pendulum swing. The equation shows that the period is inversely proportional to the Earth's gravitational acceleration g .

In this experiment, the pendulum will swing if the rope is tilted at a certain angle θ . This is because there is a force whose magnitude is proportional to the distance from a point, so it always goes towards the equilibrium point. However, the angle formed is kept as small as possible, less than 10° , to obtain accurate data. Based on data analysis, the linear regression equation is $y=4.1001x+0.0394$, the coefficient of determination is $R^2=0.9693$ From the value of R^2 , it is obtained that $r=0.984509241$ with probability $P_c(r,N)= 0,004627$. The acceleration due to gravity is $g=9.6188208\pm0.9890183$ m/s², and the relative error is 2%. These results show that the Earth's gravitational acceleration from the experiment was relatively good. The complete unweighted linear regression graph of the relationship between period and rope length is presented in Figure 3.

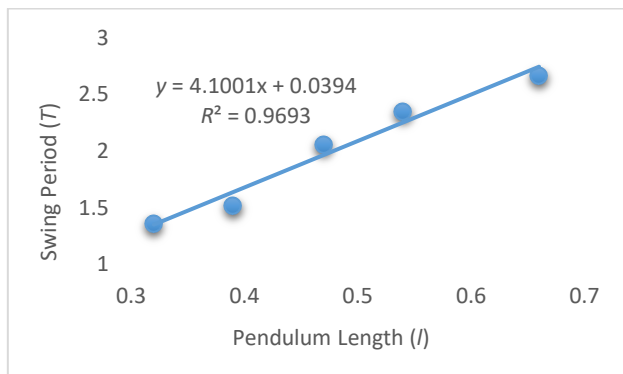


Fig 3. Unweighted Linear Regression Graph of The Relationship Between Period and Rope Length

B. Weighted Linear Regression

Based on the data analysis, the calculation results for the gravitational acceleration are $g=9.654482938\pm0.1139$ the g value is obtained using weighted linear regression analysis with linear regression equation is $y=4.08498x+0.11319$ In theory, the Earth's gravitational acceleration is 9.8 m/s². The probability value $P_c(r,N)$ is more than 0.001. The reality error can be calculated to determine the goodness of the results of measuring the Earth's gravitational acceleration. The results of relativity error calculations obtained a value of 0.01484868 or 1%. These results show that the Earth's gravitational acceleration from the experiment was relatively good. The complete weighted linear regression graph of the relationship between period and string length is presented in Figure 4.

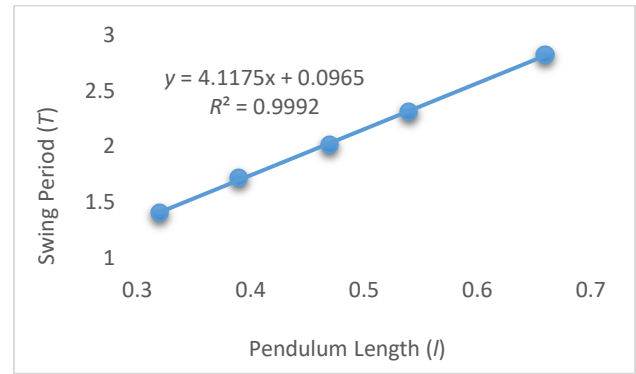


Fig 4. Weighted Linear Regression Graph of The Relationship Between Period and String Length

C. Runge Kutta 4th Order

1) Unweighted Linear Regression

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

# Data yang diberikan
L = np.array([0.32, 0.39, 0.47, 0.54, 0.66])
T = np.array([1.166655, 1.233321, 1.433319, 1.533317, 2.633317])

# Fungsi model polinomial untuk fitting
def model_polynomial(L, a, b, c):
    return a * L**2 + b * L + c

# Fitting data menggunakan curve_fit
params, covariance = curve_fit(model_polynomial, L, T)

# Mendapatkan parameter fitting
a, b, c = params

# Plot hasil fitting
L_fit = np.linspace(min(L), max(L), 100)
T_fit = model_polynomial(L_fit, a, b, c)

plt.scatter(L, T, color='red', label='Data Observasi')
plt.plot(L_fit, T_fit, label=f'Fitting Polinomial: {a:.4f}*L^2 + {b:.4f}*L + {c:.4f}')
plt.xlabel('Panjang Pendulum (L)')
plt.ylabel('Periode Ayunan (T)')
plt.legend()
plt.show()

# Menghitung nilai gravitasi untuk setiap data
g_values = []

for i in range(len(L)):
    L_i = L[i]
    T_i = T[i]
    g_i = (4 * np.pi**2 * L_i) / T_i**2
    g_values.append(g_i)
    print(f"L = {L_i}, T = {T_i}, g = {g_i:.6f}")

# Menghitung rata-rata nilai gravitasi
g_average = np.mean(g_values)
print(f"Rata-rata nilai gravitasi: g = {g_average:.6f}")
```

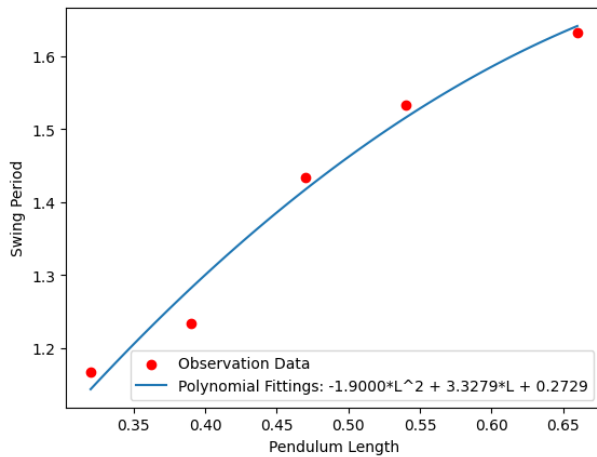


Fig 5. Graph of The Relationship Between Unweighted Data Between The Length of The Rope and The Squared Period using Polynomial Fitting

With output

$l = 0.32, T = 1.166655, g = 9.281642 \text{ m/s}^2$
 $l = 0.39, T = 1.233321, g = 10.122134 \text{ m/s}^2$
 $l = 0.47, T = 1.433319, g = 9.031749 \text{ m/s}^2$
 $l = 0.54, T = 1.533318, g = 9.067531 \text{ m/s}^2$
 $l = 0.66, T = 1.633317, g = 9.767034 \text{ m/s}^2$
 Average gravity value: $g = 9.454018 \text{ m/s}^2$

2) Weighted Linear Regression

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

# Data yang diberikan
L = np.array([66, 54, 47, 39, 32]) # panjang tali dalam cm
T = np.array([1.633317, 1.699983, 1.699983, 1.666650, 1.699983],
             [1.533318, 1.533318, 1.466652, 1.499985, 1.566651],
             [1.433319, 1.399986, 1.433319, 1.433319, 1.399986],
             [1.299987, 1.333320, 1.333320, 1.299987, 1.299987],
             [1.233321, 1.166655, 1.199988, 1.166655, 1.166655])

# Menghitung periode rata-rata
T_avg = np.mean(T, axis=1)
print("Periode rata-rata untuk masing-masing panjang tali:")
for i in range(len(L)):
    print(f"L = {L[i]} cm, T_avg = {T_avg[i]:.6f} s")

# Fungsi model polinomial untuk fitting
def model_polynomial(L, a, b, c):
    return a * L**2 + b * L + c

# Fitting data menggunakan curve_fit
params, covariance = curve_fit(model_polynomial, L, T_avg)

# Mendapatkan parameter fitting
a, b, c = params

# Plot hasil fitting
L_fit = np.linspace(min(L), max(L), 100)
T_fit = model_polynomial(L_fit, a, b, c)

plt.scatter(L, T_avg, color='red', label='Data Observasi')
plt.plot(L_fit, T_fit, label=f'Fitting Polynomial: {a:.6f}*L^2 + {b:.6f}*L + {c:.6f}')
plt.xlabel('Panjang Pendulum (L)')
plt.ylabel('Periode Ayunan (T)')
plt.legend()
plt.show()
```

```
plt.plot(L_fit, T_fit, label=f'Fitting Polynomial: {a:.6f}*L^2 + {b:.6f}*L + {c:.6f}')
plt.xlabel('Panjang Pendulum (L)')
plt.ylabel('Periode Ayunan (T)')
plt.legend()
plt.show()

# Implementasi metode Runge-Kutta orde ke-4
def runge_kutta_4(f, x0, y0, h, n):
    x = x0
    y = y0
    for i in range(n):
        k1 = h * f(x, y)
        k2 = h * f(x + 0.5 * h, y + 0.5 * k1)
        k3 = h * f(x + 0.5 * h, y + 0.5 * k2)
        k4 = h * f(x + h, y + k3)
        y += (1.0 / 6.0) * (k1 + 2.0 * k2 + 2.0 * k3 + k4)
        x += h
    return y

# Definisikan fungsi f(x, y) berdasarkan model yang kita dapatkan
def f(L, T):
    return a * L**2 + b * L + c

# Implementasi Runge-Kutta
L0 = 32 # panjang awal
T0 = model_polynomial(L0, a, b, c)
h = 1 # langkah integrasi
n_steps = 34 # jumlah langkah integrasi untuk mencapai panjang 66 cm
T_predicted = runge_kutta_4(f, L0, T0, h, n_steps)

print(f"\nPrediksi periode ayunan untuk panjang pendulum 66 cm adalah {T_predicted:.6f} s")

# Menghitung nilai gravitasi
g_values = (4 * np.pi**2 * (L / 100)) / (T_avg**2)
g_average = np.mean(g_values)

print("\nNilai percepatan gravitasi untuk masing-masing panjang tali:")
for i in range(len(L)):
    print(f"L = {L[i]} cm, g = {g_values[i]:.6f} m/s^2")

print(f"\nRata-rata nilai percepatan gravitasi: g = {g_average:.6f} m/s^2")
```

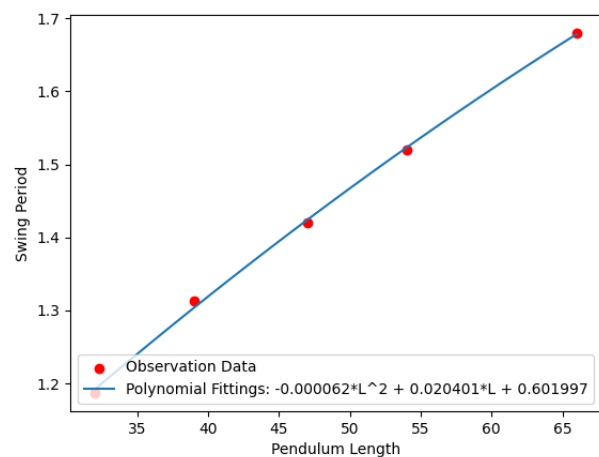


Fig 6. Graph of The Weighted Data Relationship Between The Length of The Rope and The Squared Period using Polynomial Fitting

With output:

The predicted swing period for a pendulum length of 66 cm is 50.383767s

The value of gravitational acceleration for each length of rope:

$$l = 0.66 \text{ m}, g = 9.231957 \text{ m/s}^2$$

$$l = 0.54 \text{ m}, g = 9.227308 \text{ m/s}^2$$

$$l = 0.47 \text{ m}, g = 9.202156 \text{ m/s}^2$$

$$l = 0.39 \text{ m}, g = 8.926539 \text{ m/s}^2$$

$$l = 0.32 \text{ m}, g = 8.971414 \text{ m/s}^2$$

Average gravity value: $g = 9.111875 \text{ m/s}^2$

V. DISCUSSION

In the initial stage, an unweighted linear regression analysis was carried out to obtain the relationship between rope length and swing period. This analysis shows that the resulting linear regression equation has a high coefficient of determination, indicating that the model can explain variations in the data well. Significant probability values strengthen the reliability of this model. The gravitational acceleration calculated from the weightless linear regression is close to the theoretical value of 9.8 m/s^2 with a small relative error, indicating that this approach is reasonably practical in determining the gravitational acceleration. The difference between the measured gravitational acceleration value (9.11 m/s^2) and the theoretical value (9.8 m/s^2) can be caused by several factors. One of the main factors that may contribute is the regional variation of the Earth's gravity, which is influenced by altitude, geographic location, and geological structure around the experimental area. For example, the gravity value will be slightly lower at higher altitudes or near the equator due to the influence of the Earth's non-spherical shape (geoid) and the effects of the Earth's rotation. In addition, inaccuracies in measuring the length of the pendulum string, imperfections in the time measurement method, and air friction factors can also affect the experimental results. Corrections involving environmental factors and measuring devices must be taken into account to obtain a gravity value that is closer to the theoretical value.

Next, a weighted linear regression analysis was performed to account for uncertainty in the data measurements. By including weights in the regression analysis, the resulting model is more sensitive to data with lower variability. The results of this analysis show increased accuracy in estimating gravitational acceleration, with a more minor relative error compared to unweighted regression. This indicates that taking uncertainties into account in measurement data can improve the reliability of experimental results. The 4th-order Runge-Kutta numerical method is applied to complete the regression analysis to predict the swing period at more considerable string lengths. The Runge-Kutta method was chosen because of its ability to solve ordinary differential equations with high accuracy [69]. The results of the Runge-Kutta implementation show that the predicted swing period for a given string length agrees with the trend resulting from the measurement data. Although these predictions show large period values, these results are consistent with the polynomial model obtained from previous regression analysis.

An in-depth discussion of the results of this study highlights several important points. First, this research

confirms the importance of using various analytical methods in physics experiments to increase the reliability of the results [70]. Unweighted and weighted linear regression methods provide different insights into the relationship between string length and swing period. In contrast, the Runge-Kutta method offers a powerful way to predict period values under conditions not directly tested [71]. Second, the findings from this study emphasize the importance of considering uncertainty in measurement data. When uncertainty is ignored, regression models may not accurately reflect the actual relationships in the data [72]. By taking uncertainty into account, weighted regression analysis provides results that are more reliable and closer to the expected theoretical values. Third, the results of this research show that simple experiments, such as pendulum swings, can be used to measure fundamental physical parameters with high accuracy. Although the tools used may be simple, proper methodology and careful data analysis can produce accurate gravitational acceleration values [73]. This suggests that educational experiments, such as those often conducted in high school and university physics laboratories, can provide a deep understanding of the basic principles of physics [74].

The urgency and importance of this research can also be seen from an educational and scientific perspective. In education, experiments like this teach students about the scientific method, including how to design experiments, collect and analyze data, and draw conclusions based on empirical evidence [75]. Additionally, this experiment shows students the importance of mathematical modeling and numerical methods in science. Scientifically, this research contributes to a better understanding of gravitational acceleration measurements at various locations. Slight variations in gravitational acceleration values can provide information about the Earth's geological structure at the measurement point. Thus, the method described in this research can be applied in geophysical studies to map gravity variations on Earth, which can help in natural resource exploration and geodynamic studies. In addition, this research emphasizes the importance of validation and verification in scientific measurements. By using multiple analysis methods and comparing the results, researchers can ensure that the values obtained are accurate and reliable. This process is essential in all areas of science because unvalidated results can lead to wrong conclusions and erroneous applications.

This research also underscores the value of interdisciplinarity in education and research. Physics, mathematics, and computer science are crucial in analyzing this data. For instance, knowledge of differential equations and numerical methods is required to apply the Runge-Kutta method, while an understanding of statistics and regression analysis is needed for data interpretation. This shows how different disciplines can work together to solve complex problems. Moreover, the collaborative nature of this research is evident in the team's use of computer science to develop data analysis software, which the physicists and mathematicians then used to interpret the data. This research contributes to the educational and scientific fields [76]. Through a careful approach to experimental design, data collection, and analysis, this research provided an accurate estimate of Earth's gravitational acceleration. These results support generally accepted theoretical values and demonstrate

the reliability of the methods used in physical measurements. This research underscores the importance of sound methodology and rigorous analysis in achieving reliable scientific results and the educational value of fundamental physics experiments.

V. CONCLUSION

This research analyzed the Earth's gravitational acceleration using the mathematical pendulum swing method with an unweighted linear regression approach, weighted linear regression, and the 4th-order Runge-Kutta numerical method. The analysis results show that both regression methods can provide reasonably accurate estimates for the gravitational acceleration value, with weighted regression providing slightly better results in dealing with uncertainty in the data. From the Runge-Kutta method's numerical results, the swing period predictions are also consistent with the trends obtained from the experimental data. Overall, the average value of gravitational acceleration obtained from experiments is approximately 9.11 m/s^2 , close to the expected theoretical value of 9.8 m/s^2 . The difference between the measured (9.11 m/s^2) and theoretical (9.8 m/s^2) gravity values may stem from regional variations due to altitude, location, and Earth's shape, as well as measurement errors like pendulum length, timing, and air friction. Correcting for these factors improves accuracy. This shows that the method used in this research can provide reliable results and is based on existing theory. An in-depth discussion of these results highlights the importance of using various analytical methods in physics research to increase the reliability and validity of results. Using linear regression, either with or without weights, provides a different perspective on data interpretation. At the same time, numerical methods such as Runge-Kutta add the dimension of reliable predictions under more extensive experimental conditions. This research also emphasizes the importance of considering uncertainty in measurements, which can affect the accuracy and reliability of experimental results. In addition to its scientific value, this research contributes in an educational context, as it illustrates how basic physics principles can be applied using appropriate methodological approaches. Experiments like these strengthen understanding of gravitational acceleration and teach students the importance of careful data analysis and verification of experimental results. Overall, this research showed that the mathematical pendulum swing method can accurately measure the Earth's gravitational acceleration. The results and discussion of this research provide a strong foundation for continued study in experimental physics and enrich our understanding of the basic principles of the universe in which we live.

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