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Suspension Bridge Estimation Method using the Fokker-Planck Model

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***Abstract*—The failure of the suspension bridge has been known since the beginning of the bridge collapse. Most of these failures form the basis of current engineering knowledge. One of the factors of failure is human-made factors related to the calculation of the bridge estimate. This paper presents an indirect estimation method using numerical simulation using finite elements by analyzing the Fokker-Planck model when dynamic excitation is associated with bridge loads. The results show that the Fokker-Planck model's homogeneous form can take into account the solution for the bridge analysis approach. It leads to a stable state when giving mass variations to the model. The indirect estimation method using finite elements can estimate the cable tension with controllable weak damping. It can be concluded that the method in this study is more accurate and convenient for the application technique.**

***Index Terms*—estimation method, finite element, fokker-planck model, mathematical models, weakly damped wave.**

I. Introduction

THE application of nonlinear dynamic case mathematical modeling is a suspension bridge, or it is called a nonlinear jump [1,2]. Nonlinear jumping is one type of phenomenon introduced by the wave theory [3-7]. This phenomenon is an interpretation of nonlinear differential equations and requires an approach to determine the solution of the solution [8-14]. In addition to solutions, another thing needed is a control function that can be used as a controlling estimation when there is vibration stimulation. Ref. [15] has modeled this in (equation 1).

$$m\frac{\partial^2 v(x,t)}{\partial t^2}+EI\frac{\partial^4 v(x,t)}{\partial x^4}+b\frac{\partial v(x,t)}{\partial t}+ k(u-v)^{+}=W(x)+\varepsilon f(x,t) \quad \text{..... (1)}$$

$$m_1\frac{\partial^2 u(x,t)}{\partial t^2}+T\frac{\partial^2 u(x,t)}{\partial x^2}+b_1\frac{\partial u(x,t)}{\partial t}- k(u-v)^{+}=W(x)+\varepsilon f_1(x,t)$$

with m and m_1, mass per unit length of the bridge and main cable, E modulus young, I cross-section of moment of inertia, k stiffness of cable, b and b_1 of damping coefficient on bridge deck and main cable, W and W_1 weight per unit length of the bridge and main cable, T the inner stresses of the main cable, εf and εf_1 are the external time-periodic enforcers on the main bridge and cable [16-24].

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The factors that cause the collapse of the suspension bridge are natural phenomena such as earthquakes, winds, cyclones, exploration related to the phenomenon of lower riverbeds due to water erosion, landslides, and human-made factors related to imperfect design and construction [25]. The collapse of the West Gate Bridge in Australia in 1970 was one of the design and construction methods used in negligence [26-28]. Meanwhile, overvoltage can cause bridge failures, such as the Kutai Kartanegara bridge in Indonesia in 2001 [29].

In general, there are two categories of stress estimation methods, namely, direct and indirect ways. The estimation method is directly related to cable tension, but it is an expensive method [30-32]. Meanwhile, the indirect method of solving test parameters uses computation [33]. The indirect method is related to the use of the finite element method, which is a solution to the hanging bridge model approach [34-38].

Based on the previous description, this study aims to determine the estimation method to minimize the losses and failures of the suspension bridge using the finite element method with the Fokker-Planck model.

II. Method

This research is theoretical and computational. It is carried out by programming using Matlab. The first step is to determine the parameters used to solve a suspension bridge's physical model analytically, subject to time function damping, namely defining the boundary conditions.

$$\frac{\partial u(0,t)}{\partial x}=\frac{\partial u(L,t)}{\partial x}=u(0,t)=u(L,t)=0 \quad \text{..... (2)}$$

while the initial conditions are given,

$$u(x,0)=\exp\left[-0,999(3x-1)^2\right] \quad \text{..... (3)}$$

The second stage determines the analysis of the wave equation model using the Fokker-Planck model [39,40], with the following assumptions:

1. The location of particle displacement is expressed in terms of the expected price, which is written as, $E(x)=\langle x\rangle=0$
2. The amount of particle displacement by nonzero constants, namely γ and λ, $u(x)=\frac{\gamma^2}{\lambda}$

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3. The probability of the particle at position *x* and time *t* moving to the left $\alpha(x,t)$
4. The probability of the particle at position *x* and time *t* moving to the right $\beta(x,t)$
5. The probability of the state of the particles at *x* time to $t+\tau$

The probability of the particle's state at *x* time to $t+\tau$ moving from the left is equal to the probability at the umpteenth point $x-\delta$ at time *t* to the probability *p*. Meanwhile, the displacement from the right can be determined by the particles' probability $x+\delta$ at time *t* the umpteenth time against the probability *q*. The displacement from the left can be written as

$$\alpha(x,t+\tau)=p\alpha(x-\delta,t)+q\beta(x-\delta,t) \quad \text{..... (4)}$$

and

$$\alpha(x,t+\tau)=p\alpha(x+\delta,t)+q\beta(x+\delta,t) \quad \text{..... (5)}$$

Taylor series expansion for $\alpha(x,t+\tau)$ can be expressed as the 1st order is

$$\alpha(x,t+\tau)=\alpha(x,t)+\tau\alpha_t(x,t) \quad \text{..... (6)}$$

for $\alpha(x,t-\delta,t)$ can be expressed as a 2nd order Taylor expansion

$$\alpha(x,t-\delta,t)=\alpha(x,t)-\delta\alpha_x(x,t)+\frac{1}{2}\delta^2\alpha_{xx}(x,t) \quad \text{..... (7)}$$

for $\alpha(x,t+\delta,t)$ can be stated

$$\alpha(x,t+\delta,t)=\alpha(x,t)+\delta\alpha_x(x,t)+\frac{1}{2}\delta^2\alpha_{xx}(x,t) \quad \text{..... (8)}$$

Taylor series expansion for $\beta(x,t+\delta,t)$ can be stated

$$\beta(x,t+\tau)=\beta(x,t)+\tau\beta_t(x,t) \quad \text{..... (9)}$$

for $\beta(x,t-\delta,t)$ can be stated

$$\beta(x,t-\delta,t)=\beta(x,t)-\delta\beta_x(x,t)+\frac{1}{2}\delta^2\beta_{xx}(x,t) \quad \text{..... (10)}$$

for $\beta(x,t+\delta,t)$ can be stated

$$\beta(x,t+\delta,t)=\beta(x,t)+\delta\beta_x(x,t)+\frac{1}{2}\delta^2\beta_{xx}(x,t) \quad \text{..... (11)}$$

If Equation (6), Equation (7), and (10) are substituted for Equation (4) is obtained,

$$\tau\alpha_t(x,t)=p\begin{bmatrix}\alpha(x,t)-\delta\alpha_x(x,t)+\\ \frac{1}{2}\delta^2\alpha_{xx}(x,t)\end{bmatrix}+ q\begin{bmatrix}\beta(x,t)-\delta\beta_x(x,t)+\\ \frac{1}{2}\delta^2\beta_{xx}(x,t)\end{bmatrix}-\alpha(x,t) \quad \text{..... (12)}$$

The rate $\frac{1}{2}\delta^2\alpha_{xx}(x,t)$ and $\frac{1}{2}\delta^2\beta_{xx}(x,t)$ can be ignored because it only gives a tiny contribution or value. If equation (12) for each side is divided by τ, then the equation becomes,

$$\alpha_t(x,t)=\frac{p}{\tau}\alpha(x,t)-\frac{p}{\tau}\delta\alpha_x(x,t)+ \frac{q}{\tau}\beta(x,t)-\frac{q}{\tau}\delta\beta_x(x,t)-\frac{\alpha}{\tau}(x,t) \quad \text{..... (13)}$$

or by simplifying the equation to be,

$$\alpha_t(x,t)=\frac{1}{\tau}(p-1)\alpha(x,t)-\frac{p}{\tau}\delta\alpha_x(x,t)+ \frac{q}{\tau}\beta(x,t)-\frac{q}{\tau}\delta\beta_x(x,t) \quad \text{..... (14)}$$

This applies to terms of $q/\tau\,\delta\beta_x(x,t)$, due to the acceleration from the right whose value is so small that the term can be ignored,

$$\alpha_t(x,t)=\frac{1}{\tau}\big((p-1)\alpha(x,t)-p\delta\alpha_x(x,t)\big)+ \frac{q}{\tau}\beta(x,t) \quad \text{..... (15)}$$

by using the total probability that is $p + q = 1$, then equation (15) becomes

$$\alpha_t(x,t)=-\frac{q}{\tau}\alpha(x,t)-\frac{p}{\tau}\delta\alpha_x(x,t)+\frac{q}{\tau}\beta(x,t) \quad \text{.... (16)}$$

The equation (16) can also be assumed as $\lim_{\delta\to\infty}^{\tau\to 0} q/\tau=\lambda$ for λ constants non-zero and $\lim_{\delta\to\infty}^{\tau\to 0} p\delta/\tau=\gamma$, while for γ constants non-zero. Hence, by using the commutative law, Equation (16) becomes

$$\frac{\partial\alpha}{\partial t}(x,t)=-\gamma\frac{\partial\alpha}{\partial x}(x,t)-\lambda\alpha(x,t)+\lambda\beta(x,t) \quad \text{..... (17)}$$

Equation (17) is a form of partial differential equation for Equation (4). In the same way for the structure of the differential equation for Equation (5), namely by substituting Equation (8), Equation (9), and Equation (11) to Equation (5), the final form of the equation is obtained,

$$\frac{\partial\beta}{\partial t}(x,t)=\gamma\frac{\partial\beta}{\partial x}(x,t)+\lambda\alpha(x,t)-\lambda\beta(x,t) \quad \text{..... (18)}$$

If Equations (17) and (18) are added, then

$$\frac{\partial}{\partial t}(\alpha+\beta)(x,t)+\gamma\frac{\partial}{\partial x}(\alpha-\beta)(x,t)=0 \quad \text{..... (19)}$$

if Equations (18) and (19) are subtracted, then

$$\frac{\partial}{\partial t}(\alpha-\beta)(x,t)+\gamma\frac{\partial}{\partial x}(\alpha+\beta)(x,t)+ 2\lambda(\alpha-\beta)(x,t)=0 \quad \text{..... (20)}$$

The derivative with respect to *t* Equation (20) is

$$\gamma\frac{\partial^2}{\partial x\partial t}(\alpha+\beta)(x,t)+\frac{\partial^2}{\partial t^2}(\alpha-\beta)(x,t)+ 2\lambda\frac{\partial}{\partial t}(\alpha-\beta)(x,t)=0 \quad \text{..... (21)}$$

The derivative with respect to *x* of Equation (20) is then multiplied by γ is

$$\gamma^2\frac{\partial^2}{\partial x^2}(\alpha+\beta)(x,t)+\gamma\frac{\partial^2}{\partial x\partial t}(\alpha-\beta)(x,t)+ 2\lambda\gamma\frac{\partial}{\partial x}(\alpha-\beta)(x,t)=0 \quad \text{..... (22)}$$

add the Equations (21) and (22)

$$\frac{\partial^2}{\partial t^2}(\alpha+\beta)(x,t)+\gamma^2\frac{\partial^2}{\partial x^2}(\alpha+\beta)(x,t)+ 2\alpha\gamma\frac{\partial}{\partial x}(\alpha-\beta)(x,t)+2\alpha\gamma\frac{\partial}{\partial t}(\alpha-\beta)(x,t)=0 \quad \text{..... (23)}$$

if assumed $u(x,t)=\alpha(x,t)\pm\beta(x,t)$, so that Equation (23) becomes,

$$\frac{\partial^2}{\partial t^2}u(x,t)+\gamma^2\frac{\partial^2}{\partial x^2}u(x,t)+ 2\lambda\gamma\frac{\partial}{\partial x}u(x,t)+2\lambda\gamma\frac{\partial}{\partial t}u(x,t)=0 \quad \text{..... (24)}$$

Equation (24) is the general form of the Fokker-Planck model. Assuming that $m_1=1$, $\gamma^2=T$, $2\lambda\gamma=b_1$ and the damping factor with respect to distance is considered zero, then Equation (24) can be written,

$$m_1\frac{\partial^2}{\partial t^2}u(x,t)+T\frac{\partial^2}{\partial x^2}u(x,t)+b_1\frac{\partial}{\partial t}u(x,t)=0 \quad \text{..... (25)}$$

The third stage is to determine the system stability requirements by providing mass variations in the Fokker-Planck model.

II. Results and Discussion

The collapse of the suspension bridge is a critical point for developing scientific disciplines regarding modeling. It is in line with the many studies on improving the suspension bridge model. One of the modeling alternatives is to provide weak attenuation, following the Brownian Motion Backward Kolmogorov or Fokker-Planck models. The solution is shown in (equation 26),

$$m_1\left(\frac{1}{T_{(t)}}\frac{d^2T_{(t)}}{dt^2}\right)+T_1\left(\frac{1}{X_{(x)}}\frac{d^2X_{(x)}}{dx^2}\right)+ b_1\left(\frac{1}{T_{(t)}}\frac{dT_{(t)}}{dt}\right)=0 \quad \text{..... (26)}$$

by taking certain constants $-k^2$, to represent a time constant $T_{(t)}$ and $-n^2$ to represent distance constants $X_{(x)}$ obtained,

$$m_1k^2-T_1n^2+b_1Ae^{k^2t}=0 \quad \text{..... (27)}$$

with

$$k^2=T_{(t)_{\text{orde-2}}}\begin{cases}e^{kt}\\ e^{-kt}\end{cases},\ -n^2=X_{(x)}\begin{cases}e^{ix}\\ e^{-ix}\end{cases},\ T_{(t)_{\text{orde-1}}}=Ae^{k^2t} \quad \text{..... (28)}$$

Equation (26) is a unique form of the Fokker-Planck homogeneous model. If the form is non-homogeneous then

$$m_1k^2-T_1n^2+b_1Ae^{k^2t}=1 \quad \text{..... (29)}$$

The non-homogeneous form is unique in that it represents a force of force that estimates the influence from outside the system. It is related to Equation (3), namely the weight per unit length of the bridge and the external time-periodic force on the main bridge and cable. In its solution, Equation (27) and Equation (29) are easier to solve using the finite element method [41,42] with definitions,

$$\frac{\partial u(x,t)}{\partial x}=u(x_j,t_k)=u_j^k \quad \text{..... (30)}$$

The transformation of the second derivative center difference with respect to t is

$$\frac{\partial^2u(x,t)}{\partial t^2}=u_{tt}\left(x_j,t_{k+\frac{1}{2}}\right)=\frac{u_j^{k+1}-2u_j^{k+\frac{1}{2}}+u_j^k}{\Delta t^2} \quad \text{..... (31)}$$

The transformation of the second derivative's center difference with respect to x is

$$\frac{\partial^2u(x,t)}{\partial x^2}=u_{xx}\left(x_{j+\frac{1}{2}},t_k\right)=\frac{u_{j+1}^k-2u_{j+\frac{1}{2}}^k+u_j^k}{\Delta x^2} \quad \text{..... (32)}$$

while the second derivative forward difference transformation with respect to t is

$$\frac{\partial u(x,t)}{\partial t}=u_t(x_j,t_{k+1})=\frac{u_j^{k+1}-u_j^k}{2\Delta t} \quad \text{..... (33)}$$

then the discrete form of equation (27) for the finite element transformation is

$$m_1\frac{u_j^{k+1}-2u_j^{k+\frac{1}{2}}+u_j^k}{\Delta t^2}+T\frac{u_{j+1}^k-2u_{j+\frac{1}{2}}^k+u_j^k}{\Delta x^2}+ b_1\frac{u_j^{k+1}-u_j^k}{2\Delta t}=0 \quad \text{..... (34)}$$

Equation (34) is a form of finite element transformation. Gan [33] stated that the finite element method effectively estimates the tension of the hanger cables with a minimum relative error of less than 3%. However, what distinguishes this study is the addition of a weak damping factor. The result of algebraic manipulation for the iteration formula in Equation (34) produces

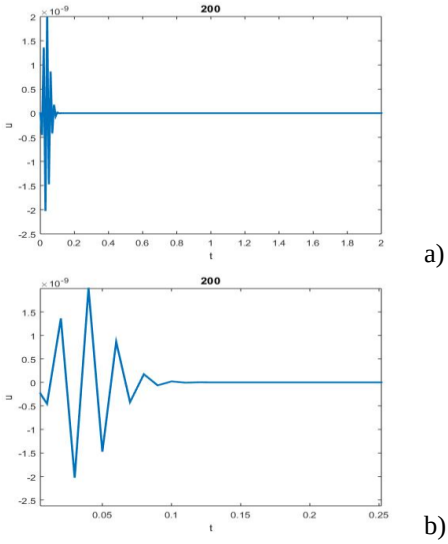
$$u_j^k=-\frac{2m_1}{(b_1\Delta t+2m_1)}\left(-2u_j^{k-\frac{1}{2}}+u_j^{k-1}\right)- \lambda\frac{2\Delta t}{(b_1\Delta t+2m_1)}\left(u_{j+1}^{k-1}-2u_{j+\frac{1}{2}}^{k-1}+u_j^{k-1}\right)+ \frac{b_1\Delta t}{(b_1\Delta t+2m_1)}u_j^{k-1} \quad \text{..... (35)}$$

Meanwhile, the iteration for the non-homogeneous Fokker-Planck model is

$$u_j^k=\frac{2\Delta t^2}{(b_1\Delta t+2m_1)}-\frac{2m_1}{(b_1\Delta t+2m_1)}\left(-2u_j^{k-\frac{1}{2}}+u_j^{k-1}\right)- \lambda\frac{2\Delta t}{(b_1\Delta t+2m_1)}\left(u_{j+1}^{k-1}-2u_{j+\frac{1}{2}}^{k-1}+u_j^{k-1}\right)+ \frac{b_1\Delta t}{(b_1\Delta t+2m_1)}u_j^{k-1} \quad \text{..... (36)}$$

If a value is selected $\Delta t = 0.01$, $\Delta x = 0.25$, and $g = 9.8$ then the value $\lambda=T_1\Delta t/\Delta x^2=784$ where $T=m_1g$ so that Equation (35) can be written,

$$u_j^k=-\frac{2m_1}{(b_1\Delta t+2m_1)}\left(-2u_j^{k-\frac{1}{2}}+u_j^{k-1}\right)- 784\frac{2\Delta t}{(b_1\Delta t+2m_1)}\left(u_{j+1}^{k-1}-2u_{j+\frac{1}{2}}^{k-1}+u_j^{k-1}\right)+ \frac{b_1\Delta t}{(b_1\Delta t+2m_1)}u_j^{k-1} \quad \text{..... (37)}$$

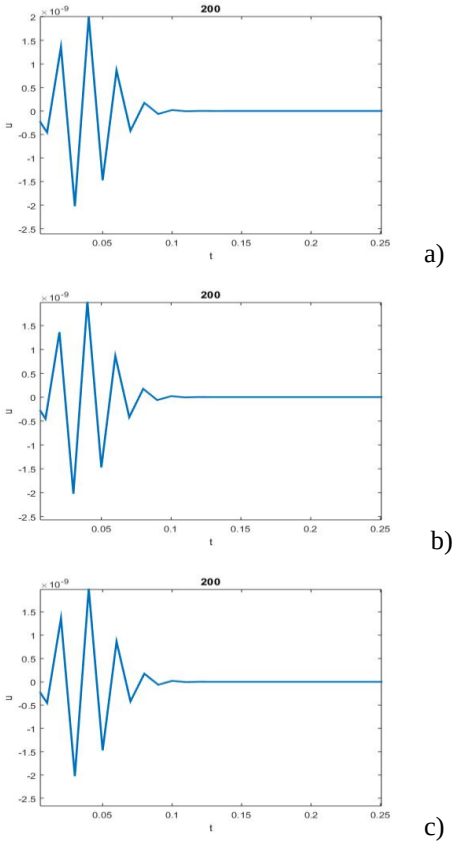


a)

b)

Fig. 1. a). Fokker-Planck homogeneous model and b). Magnification result with *m*=500.

Fig. 1 shows that the homogeneous Fokker-Planck model with mass variations shows a stable condition. Ref. [15] conducted these results regarding a durable bridge response to vertical motion with the 4th-order Runge-Kutta method using the SIMULINK scheme. Meanwhile, Ref. [12] explains that for the settlement of considerable energies in bridge structures, the thing to pay attention to is to provide attenuation.



a)

b)

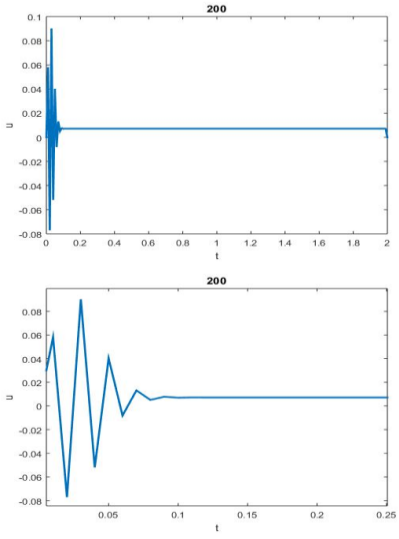
c)

Fig. 2. Fokker-Planck homogeneous model and magnification result (a) *m*=5000; (b) *m*=10000; and (c) *m*=50000.

Mass variations for the Fokker-Planck homogeneous model and the enlargement results are shown in Fig. 2. If a value is selected Δt = 0.01, Δx = 0.25, and g = 9.8 then the value $\lambda = T_1 \Delta t / \Delta x^2 = 784$ where $T=m_1 g$ so that Equation (36) can be written,

$$u_j^k = \frac{2\Delta t^2}{\left(b_1 \Delta t + 2m_1\right)} - \frac{2m_1}{\left(b_1 \Delta t + 2m_1\right)}\left(-2u_j^{k-\frac{1}{2}} + u_j^{k-1}\right) - 784\frac{2\Delta t}{\left(b_1 \Delta t + 2m_1\right)}\left(u_{j+1}^{k-1} - 2u_{j+\frac{1}{2}}^{k-1} + u_j^{k-1}\right) + \frac{b_1 \Delta t}{\left(b_1 \Delta t + 2m_1\right)} u_j^{k-1} \quad \text{.....(38)}$$

where Fig. 3 shows the simulation form of Equation (36).

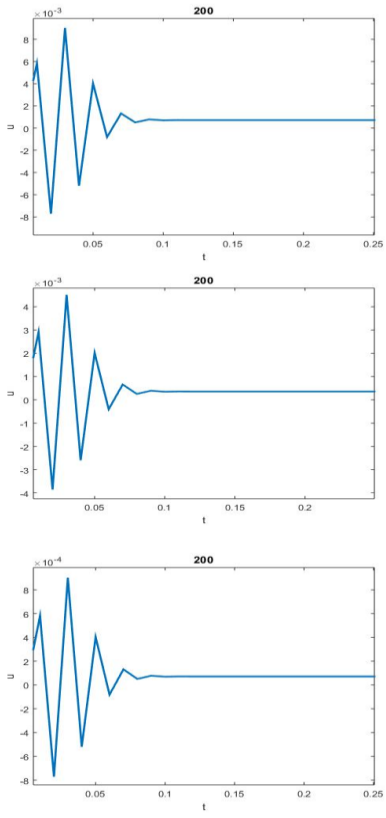


a)

b)

Fig. 3. a). Fokker-Planck non-homogeneous model and b). Magnification result with *m*=500.

As a comparison, Fig 4 shows an image based on the variation of *m*=5000, *m*=10000, and *m*=50000.



a)

b)

c)

Fig. 4. Fokker-Planck non-homogeneous model and magnification result (a) *m*=5000; (b) *m*=10000; and (c) *m*=50000.

The change in the *y*-axis of magnification with *m*=500 for the homogeneous and non-homogeneous Fokker-Planck models is shown in Fig. 5.

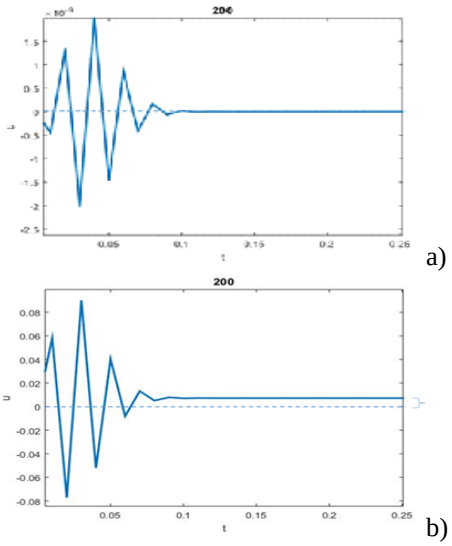


Fig. 5. (a) Fokker-Planck homogeneous model and (b) Fokker-Planck non-homogeneous model.

The differences between the homogeneous and non-homogeneous Fokker-Planck models based on the variations in m are presented in Table 1.

TABLE I
DIFFERENCE BETWEEN HOMOGENEOUS AND NONHOMOGENEOUS FOKKER-PLANCK SYSTEMS

Parameter	Homogeneous	Non-homogeneous
Balance	Stay in the initial position	A shift from the starting point
Mass variation	Fixed amplitude	Amplitude changes
Stable time	$0.1 \leq t \leq 2$ s	$0.1 \leq t \leq 2$ s

This study presents a more practical approach for calculating the estimated cable tension. The results of the Fokker-Planck homogeneous model's visualization produce waves with a constant amplitude of 1×10^{-8} for each time change. At $0\leq t\leq0.1$ the displacement amplitude changes, with a maximum value of 2×10^{-9}, and is stable at $0.1\leq t\leq2$. Under these conditions, the homogeneous Fokker-Planck model shows a stable condition if there is mass variation. Besides, the homogeneous model does not experience a shift from the equilibrium point. Meanwhile, the non-homogeneous model visualization results show that at $0\leq t\leq0.1$, the displacement amplitude changes and experiences an unstable condition if there is mass variation. However, when $0.1\leq t\leq2$, the system again shows a stable condition but has shifted from its equilibrium point.

V. CONCLUSION

In this work, we propose the Fokker-Planck model as a suspension bridge modeling solution. By considering the location, size, and likelihood of moving particles, we obtain two Fokker-Planck models that resemble the suspension bridge models proposed by previous researchers, namely homogeneous and non-homogeneous shapes. The uniqueness of the solutions we get requires another approach to solve the complexity of analytical solutions. We are also challenged to vary some very dominant parameters such as mass, stress, and weak damping effect as a control function of the model. The analytic solution's complexity is caused by the damping factor of the time function of the model being offered. So to approach this, we need another solution as an alternative to solving the problem. In this case, we use an indirect estimation method using finite elements. The results showed that the Fokker-Planck model's homogeneous form could be used as an alternative solution to the suspension bridge because the results obtained are more stable and controlled when there are variations in mass

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