



Exploring Pre-service Elementary School Teachers' Conceptual Understanding and Problem-Solving Strategies in Basic Mathematical Concepts

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ARTICLE INFO

Article history

Received 18 June 2026

Revised 02 September 2026

Accepted 05 September 2026

ABSTRACT

This study aims to explore pre-service elementary school teachers' conceptual understanding and problem-solving strategies in fundamental mathematics concepts, with a specific focus on arithmetic series and fractions with mixed-methods methodology. A total of 84 pre-service teachers completed a written test covering 10 basic mathematics concepts, and the scores were analyzed to identify the most difficult concepts. Based on these results, six participants representing the high, medium, and low score categories were interviewed to investigate their conceptual reasoning and to analyze their problem-solving strategies using Polya's steps. The findings reveal that arithmetic series and fractions were the most challenging concepts, as evidenced by low scores, recurring errors, and persistent misconceptions. Interview results, particularly through the lens of Polya's framework, show that high-performing participants tended to use structured procedural strategies but often failed at the looking-back stage. In contrast, low-performing participants relied on intuitive, misaligned strategies, indicating weaknesses across multiple Polya steps. Overall, this study highlights the critical need to strengthen pre-service teachers' conceptual understanding through instructional approaches that emphasize representation, reasoning, and the systematic application of problem-solving heuristics.

Keywords

Conceptual Understanding

Polya

Pre-service Teachers

Problem-Solving Strategies

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Introduction

Mathematics is a field of science that has a close relationship among operations, concepts, systems, and structures. Mathematics is called a cumulative and sequential discipline, because it has a structure and some concepts build on each other, where previous concepts and knowledge form the basis for subsequent concepts and knowledge [1]. In basic mathematical concepts, proof plays an essential role in developing analytical thinking, reasoning skills, and problem-solving [2]. In basic mathematics learning, several core topics are often categorized as basic mathematical concepts, including number and fraction operations, basic patterns and algebra, and sequence/series concepts that form the basis for solving subsequent mathematical problems. Curriculum documents and educational studies emphasize the importance of mastering these topics as a foundation for learning mathematics at the elementary school level [3].

Pre-service teachers play an essential role in fostering intellectual growth and improving students' mathematical competence. The importance of deepening mathematical performance among pre-service teachers needs to be emphasized, such as concerns about mathematical knowledge for teachers who are currently teaching, poor performance in learning mathematics is related to their willingness and habits in learning mathematical concepts, the lack of academic qualifications in student mathematics teachers which can lead to a lack of understanding of the material being taught, thus impacting on decreasing student performance in learning mathematics [4].

Many students still struggle to solve context-based math problems. To address this issue, pre-service elementary school teachers play a crucial role in improving students' understanding of mathematical concepts and problem-solving skills. Teaching students problem-solving skills is essential in the mathematics curriculum. Critical thinking, creative thinking, and conceptual reasoning are all ways to develop problem-solving skills to improve math proficiency and advance 21st-century skills. Therefore, pre-service elementary school teachers must be proficient in solving math problems because they will be responsible for re-teaching basic math concepts to young learners [5].

One obstacle pre-service teacher face in problem-solving is understanding context-based mathematics problems. This obstacle is something that needs to be considered because teachers need to implement strategies to improve problem-solving for students [6]. Previous studies have established that teachers and pre-service teachers may experience difficulties in developing deep conceptual understanding of fundamental mathematical concepts, particularly fractions [7]–[8]. Other research has also examined teachers' strategic competence when solving multistep mathematical problems [9]. However, these studies have primarily examined specific mathematical domains or particular forms of teacher knowledge, leaving

less attention to how pre-service elementary teachers' conceptual understanding and problem-solving strategies interact across a broader range of fundamental mathematical concepts. Moreover, limited attention has been given to examining how these difficulties are manifested across the stages of the problem-solving process, particularly through Polya's framework, and how the use of problem-solving strategies differs according to pre-service teachers' overall level of mathematical performance.

Furthermore, pre-service teachers' limited understanding of algebraic concepts or algebraic thinking also hinders their learning. These algebraic concepts encompass the use of other variables, such as structures, the study of relationships, and procedures for solving specific problems [10]. Another obstacle faced by pre-service teachers is the concept of the convergence of numerical or arithmetic series. Understanding these concepts is crucial for mathematical development and prepares students for higher education [11]. Furthermore, teachers often teach topics such as permutations and combinations as mere memorization of formulas, preventing them from understanding or identifying students' conceptual errors [12].

To better understand how pre-service teachers approach mathematical problems, it is important to examine not only whether they obtain correct answers but also how they construct and evaluate their solutions. Polya's problem-solving framework provides a systematic approach for examining this process through four stages: understanding the problem, devising a plan, carrying out the plan, and looking back [13]. The framework enables researchers to identify where difficulties occur during problem solving, whether in interpreting the problem, selecting an appropriate strategy, executing mathematical procedures, or verifying the reasonableness of the solution [14]. For pre-service elementary teachers, examining these stages is particularly relevant because their ability to reason through mathematical problems is closely related to their future capacity to guide elementary students in developing conceptual understanding and problem-solving skills.

Solving mathematical problems, both directly and indirectly, requires the use of mathematical techniques, concepts, and processes. Developing higher-order thinking skills is a crucial aspect of preparing pre-service teachers to meet the demands of 21st-century mathematics learning. Teachers play an essential role in shaping mathematical skills and can significantly improve student achievement with long-term impacts [15]. As a good teacher, especially in deepening mathematics, it is necessary to master the depth of content and pedagogical knowledge [16]. Being a teacher requires competence in posing problems at a higher level than simply understanding, remembering, and applying them. Higher levels include logical thinking, critical thinking, and reasoning, which are basic daily life skills [17].

Critical thinking is the ability to develop problem-solving techniques, emphasize evidence, draw logical conclusions, and obtain and process accurate information. Critical

thinking is not an innate ability, but a skill that can be learned and developed over time [18]. Critical thinking must be systematically instilled in individuals and can be practically modified. Critical thinking is considered a top priority in education, requiring greater attention in every educational process [19].

Various forms of high-level thinking include critical thinking skills that include analysis, evaluation, and assessment based on evidence and logical arguments, which form the basis for multiple forms of specific thinking [20]. It is crucial to train pre-service teachers who can keep up with the times, who generate innovation, who have the will and attitude to distinguish between right and wrong, who use scientific and analytical thinking skills, in short, who have the habit of critical thinking. Because the habit of critical thinking and the level of teacher skills play a key role in determining what extent pre-service teachers will provide critical thinking skills to their students in the future [21].

Besides critical thinking, mathematical thinking is a dynamic process that can facilitate us in understanding complex structures by combining our ideas [22]. Mathematical thinking is considered necessary because it helps students develop logical, orderly thinking and the ability to evaluate situations, which is the basis for the development of high-level intellectual abilities [23]. Mathematical thinking is basic skill students are expected to acquire in mathematics. Teachers and pre-service teachers must have comprehensive knowledge of how to teach mathematics and thinking skills to create a learning environment, so that they can develop students' mathematical thinking skills [24]. In this context, critical thinking skills are an essential foundation, because mathematical thinking not only requires understanding concepts but also evaluating, reasoning, and making the right decisions in the problem-solving process.

Based on various previous studies, most studies have focused more on conceptual difficulties and the interpretation of students' answers [25]. Although these findings provide insight into the difficulties pre-service teachers experience, these studies have not examined how pre-service teachers demonstrate their unique ways of thinking and strategies for solving basic mathematical concept problems [26]. In fact, analyzing pre-service teachers' unique ways of thinking can provide in-depth insights into the mathematical thinking process and how they construct conceptual understanding [27]. There is not much research examining approaches to solving mathematical problems among pre-service teachers, especially in the context of elementary education in Indonesia.

The urgency of this research lies ensuring that pre-service teachers possess a strong conceptual understanding and effective problem-solving skills before entering the teaching profession [28]–[29]. While numerous studies have highlighted students' conceptual errors, limited research has examined how pre-service teachers think, reason, and apply problem-

solving strategies when solving fundamental mathematics problems. Analyzing their problem-solving processes is crucial for identifying not only whether they arrive at correct solutions but also how they understand problems, formulate solution plans, execute mathematical procedures, and evaluate their answers. In this context, Polya's problem-solving framework provides a systematic lens for examining these processes through four stages: understanding the problem, devising a plan, carrying out the plan, and looking back [30]. Examining pre-service teachers' problem-solving strategies through these stages can therefore provide a more in-depth understanding of their conceptual difficulties and the ways in which they approach fundamental mathematical problems

Based on the gap analysis, this study aims to examine the difficulties pre-service teachers face in solving basic mathematical concept problems. Specifically, this study attempts to answer the following questions: (1) Do pre-service teachers experience difficulties in solving basic mathematical concept problems? (2) What are the strategies of pre-service teachers in reflecting on the conceptual understanding of basic mathematical concepts?

By analyzing pre-service teacher' responses to basic mathematical concept questions, this study is expected to contribute to understanding their mathematical thinking processes. Furthermore, the results are expected to serve as a basis for developing learning strategies and training pre-service teachers to better cultivate critical thinking and mathematical problem-solving skills in the future.

Material and Methods

This study uses a mixed-methods sequential explanatory design. This design begins with the collection and analysis of quantitative data, then continues with qualitative data to deepen and explain the quantitative findings [31]. In the first stage, quantitative data were collected through a written test containing basic mathematical concept questions to assess the difficulty level of pre-service teacher. The second stage continued with in-depth interviews with several participants selected based on the test results. These interviews aimed to explore further the strategies and specific difficulties students experience when solving these questions. The purpose of this study was to analyze the ability of pre-service teachers in solving problems in the basic mathematical concept course in the odd semester.

This approach allows researchers to obtain a comprehensive picture of students' conceptual difficulties, not only from quantitative results but also from qualitative explanations that strengthen data interpretation. The results of this study are expected to provide a deeper understanding of the characteristics of pre-service teacher' difficulties in understanding and solving basic mathematical concepts.

The participants in this study were 84 male and female pre-service teachers selected through total sampling. Data include gender, region of origin, school of origin, last major, year

of graduation, and age. In general, the majority of participants were female (88.1%), while males were 11.9%. The majority of participants were from Java and were high school graduates majoring in natural and social sciences. Based on graduation year, most participants graduated in 2025. In terms of age, participants were predominantly 19-year-old students. This study was conducted from September to November 2025 at a state university in Yogyakarta. Table 1 presents the demographic information of the study participants.

Table 1. Demographic Information (n=84)

Sample Demographics	Percentage	Sample Demographics	Percentage
Gender		Final Major	
Man	11.9	Natural science	42.86
Woman	88.1	Social science	40.48
Region of Origin		Language	1.19
Sumatra Island	1.19	Culinary art	1.19
Bangka Belitung Islands	1.19	Fashion	1.19
Java Island	95.24	Visual communication design	1.19
Kalimantan Island	1.19	Independent curriculum	1.19
Papua Island	1.19	Creative batik and textile crafts	1.19
Educational Background		Accountancy	3.57
Senior high school	77.38	Age	
Islamic high school	14.29	17 years old	8.33
Vocational high school	8.33	18 years old	55.95
Year of Graduation		19 years old	34.52
2024	10.71	20 years old	1.19
2025	82.29		

Following the demographic description of the participants, the qualitative phase involved the purposive selection of six pre-service teachers from the 84 participants who completed the quantitative test. The selection was conducted after the quantitative data had been analyzed to identify participants who could provide deeper explanations of the patterns observed in their test responses.

Following the quantitative phase, six pre-service teachers were purposively selected for the qualitative phase. The participants were selected to represent variation in overall test performance and problem-solving approaches. Two participants were selected to represent each performance level, namely higher-performing participants (PST 1 and PST 2, scores of 80 and 76), middle-performing participants (PST 3 and PST 4, scores of 67 and 66), and lower-performing participants (PST 5 and PST 6, scores of 42 and 27). Purposive selection is appropriate in qualitative research when participants are intentionally selected because they can provide rich and relevant information related to the research phenomenon [32].

The selection was based not only on the participants' overall test scores but also on the uniqueness and variation of their written problem-solving strategies. In particular, participants were selected when their responses showed contrasting ways of understanding the problem, devising a solution, carrying out procedures, or dealing with errors. This approach allowed the qualitative phase to capture diverse problem-solving patterns across

different levels of performance. Two participants from each performance level were selected to provide information-rich cases for the interview phase. The interviews were conducted to obtain deeper explanations of the reasoning underlying the participants' written solutions and to elaborate on the patterns identified in the quantitative results. This procedure is consistent with the sequential explanatory mixed-methods design, in which qualitative data are collected after the quantitative phase to help explain and elaborate quantitative findings [31].

The research instrument used was a cognitive test. The basic mathematical concept test instrument used refers to the indicators shown in Table 2.

Table 2. Cognitive test instruments

Chapter	Indicators	Chapter	Indicators
Venn Diagrams	Pre-service teachers can convert data from three food categories into a Venn diagram of three sets and calculate the members of each set using set operations.	Fractions	Pre-service teachers can explain the results of $0 \div 2$ and $0 \div 0$ mathematically and realistically.
Sets	Pre-service teachers can provide two examples of real events that produce an empty set.	Arithmetic Sequences	Pre-service teachers can calculate social arithmetic in the form of alms based on starting salary, monthly salary increases, and percentages using arithmetic series.
Algebra	Pre-service teachers can construct a perimeter model from length and width in algebraic form, create inequalities ≤ 140 , and determine the appropriate x-value.	Arithmetic Series	Pre-service teachers can recognize number patterns in a series and choose a strategy to calculate the total number or a particular term in the series.
Linear Inequalities in One Variable	Pre-service teachers can solve linear inequalities with one variable and determine their solution sets	Combinations	Pre-service teachers can count the number of handshake pairs between two groups using the simple combination concept.
Quadratic Equations	Pre-service teachers can form quadratic equations from the context of a garden's area and determine the length and width measurements that meet the conditions.	Permutations	Pre-service teachers can determine the number of circular seating arrangements that satisfy certain conditions using the concept of permutations.

The test instrument used in this study was first evaluated for validity to ensure that each item measured the intended concept. Validity testing was conducted using the Pearson product-moment correlation coefficient with SPSS. Validity was determined by the relationship between each item's score and the total test score. An item is declared valid if the significance value is below 0.05, indicating a significant relationship between the item score and the overall score. Conversely, if the significance value is above 0.05, the item is declared invalid because it does not have an adequate relationship with the construct being measured [33].

Data analysis in this study was conducted in two stages. In the first stage, quantitative data from the test results were analyzed descriptively using simple statistics (mean, median, mode, and standard deviation), and a score graph of the answer results was used to determine the level of student mastery of each basic mathematical concept. The test was administered to 84 pre-service teachers, and the results were assessed using a rubric. The highest score for each item was 10, and the lowest score was 0. Therefore, the maximum total scores each pre-service teacher could obtain was 100. The assessment rubric is shown in Table 3.

Table 3. Assessment rubric for each question

Assessment Aspects	Criteria Description	Maximum Score
Concept Understanding	Pre-service teachers demonstrate conceptual understanding, depending on the type of question, and do not make conceptual errors.	4
Accuracy of Completion Steps	The steps are carried out sequentially and according to mathematical procedures, including the correct formula.	3
Accuracy of Calculation	Accurate arithmetic operations	2
Final Answer	The final result is correct and consistent with the question's context.	1
Total Score		10

All constructed-response answers were scored by one assessor using the scoring rubric described above. To analyze how pre-service teachers apply problem-solving strategies in solving basic mathematical concept problems, the answers were analyzed using Polya's step-by-step concept [34]–[36]. Indicators for the stepwise analysis of Polya's mathematical problem-solving strategy are shown in Table 4.

Table 4. Polya's problem-solving strategy analysis indicators

No	Polya's Steps	Indicator	Description
1	Understanding the Problem	Pre-service teachers demonstrate conceptual understanding, depending on the type of question, and do not make conceptual errors.	Identifying essential information in questions
		Understand the context of the question correctly	Understand the meaning of questions correctly without misinterpreting symbols or concepts
2	Devising a Plan	Applying the right concept	Using relevant and correct concepts
		Determine the solution method based on mathematical principles	Choosing the right strategy according to the concept without showing misconceptions
3	Carrying Out the Plan	Correct calculation	Logical solution steps
		Consistent with the original plan	Perform procedures correctly without miscalculations, skipped steps, or inconsistencies.
4	Looking Back	Recheck the results	Review the answers to ensure the results make sense and align with the solution steps

No	Polya's Steps	Indicator	Description
		Perform verification	Double-check calculations, procedures, or substitutions to ensure there are no errors.
		Assess the reasonableness of the answer	Review the answer results and ensure they make sense.

The next step, after analyzing the test findings, was to ask six pre-service teachers to participate as informants in interviews. The interviews were conducted to validate the test results and ensure their validity. The interview data were then analyzed through coding, categorization, and theme development to identify the reasons for correct and incorrect answers, the problem-solving strategies used, and the conceptual barriers participants experienced. The results of this analysis were combined with the test analysis to obtain a comprehensive picture of the pre-service teachers' understanding of basic mathematical concepts.

Results and Discussions

A. Instrument Validity Test Results

Before analyzing pre-service teacher responses, a validity test was conducted to ensure that each item was appropriate and accurately measured the basic mathematical concepts focused on in the study. The results of this validity test are presented in Table 5.

Table 5. Results of the validity test of the test instrument

No	Item	N	Pearson Correlation (r)	Sig. (2-tailed)	Information
1	Venn Diagrams	84	0.628	<0.001	Valid
2	Sets	84	0.523	<0.001	Valid
3	Algebra	84	0.620	<0.001	Valid
4	Linear Inequalities in One Variable	84	0.777	<0.001	Valid
5	Quadratic Equations	84	0.568	<0.001	Valid
6	Fractions	84	0.555	<0.001	Valid
7	Arithmetic Sequences	84	0.591	<0.001	Valid
8	Arithmetic Series	84	0.629	<0.001	Valid
9	Combinations	84	0.637	<0.001	Valid
10	Permutations	84	0.588	<0.001	Valid

The validity test results show that all test items have significant correlations with the total score ($p < 0.001$). An item is declared valid if its correlation coefficient value is greater than 0.30 and is statistically significant ($p < 0.05$). Thus, all items in this instrument are deemed valid because they have correlation coefficients greater than 0.30 and are statistically significant at the 0.01 level. This indicates that each test item contributes to measuring pre-service teachers' understanding of basic mathematical concepts in elementary school. This also suggests that the difficulties encountered in solving the questions are not attributable to problems with the instrument's validity but may reflect limitations in pre-service teachers'

conceptual understanding. To further assess the consistency of the instrument, reliability analysis was conducted using Cronbach's alpha based on the scores of the 10 constructed-response items from 84 pre-service teachers. The analysis yielded a Cronbach's alpha coefficient of 0.789, indicating adequate internal consistency of the instrument.

B. Answer Analysis Results

In analyzing the answers of pre-service teacher working on basic mathematics concept problems, the results were summarized in a table and then analyzed descriptively. The results of the pre-service teachers' responses to basic mathematics concept problems are shown in Table 6.

Table 6. Results of pre-service teachers

No	Item	N	True	False	Total Score	Mean	Standard Deviation
1	Venn Diagrams	84	20	64	458	5.45	3.18
2	Sets	84	38	46	524	6.23	3.98
3	Algebra	84	17	67	360	4.28	3.61
4	Linear Inequalities in One Variable	84	17	67	446	5.30	3.64
5	Quadratic Equations	84	1	83	231	2.75	1.53
6	Fractions	84	2	82	219	2.60	1.82
7	Arithmetic Sequences	84	34	50	504	6	3.75
8	Arithmetic Series	84	4	80	209	2.48	2.15
9	Combinations	84	33	51	459	5.46	4.09
10	Permutations	84	3	81	222	2.64	1.92
Maximum Score		8,400	Total Score		3,574		
			Mean		42.54		
			Standard Deviation		18.58		

The total score of 84 pre-service teachers was 3,574, with a maximum total of 8,400 and an average of 42.54. Based on test results from 84 pre-service teachers, it can be seen that mastery of basic mathematical concepts varied across topics. Some topics showed a relatively good level of understanding, while others showed low achievement by participants. The topic of sets had the highest average of 6.23, indicating that most participants had sufficient mastery of the concept. Followed by Venn diagrams and combinations, each of which also had a relatively higher achievement score compared to the other materials.

Conversely, some materials demonstrated a low level of mastery. Arithmetic series and fractions had relatively low average scores of 2.48 and 2.60, respectively. This indicates that participants still experienced difficulty applying the concepts of arithmetic series and fraction operations. The following figure is a distribution of the average value of answers to basic mathematical-concept questions, which is illustrated in the bar chart in Figure 1.

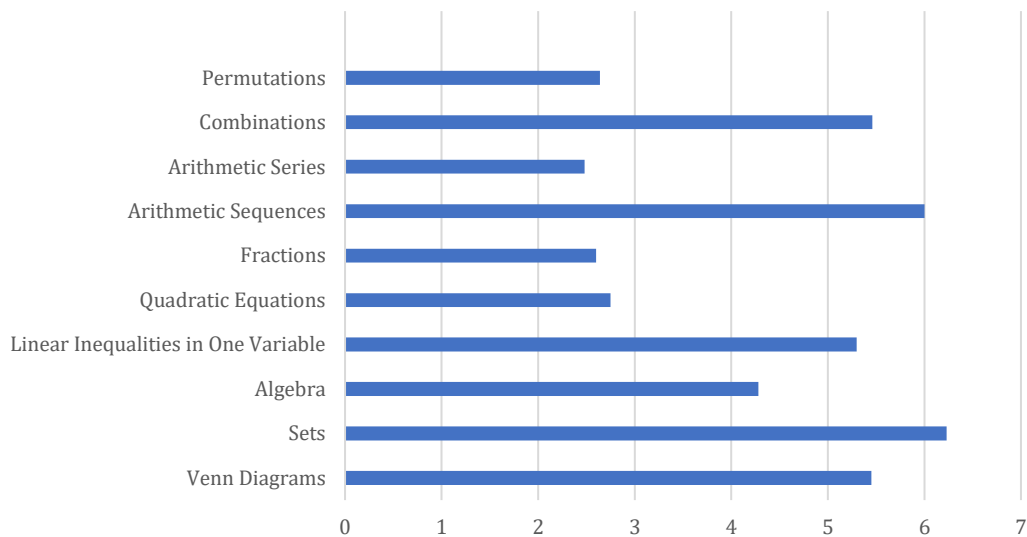


Fig. 1. A graphic image of the distribution of average answer values

Based on the graph of average answer scores, it can be seen that pre-service teachers' ability to complete 10 basic mathematics topics shows substantial variation. The topic with the highest average is sets, with an average of 6.23, indicating strong mastery of basic set operations and concepts. High achievement is also evident in arithmetic sequences, with an average score of 6, indicating that most participants can understand their patterns and formulas. In addition, the combinations and Venn diagrams obtained relatively high averages, 5.46 and 5.45, respectively, indicating a reasonably good understanding of basic combinatorial material and the representation of sets in diagrams.

In the middle-performing participants, there were linear inequalities with one variable (average 5.30) and algebra (average 4.28), indicating that some participants were able to solve problems on these two topics, even though there were still quite significant errors. Meanwhile, four topics showed significantly lower average scores. Quadratic equations (2.75), fractions (2.60), permutations (2.64), and especially arithmetic series (2.48) were the topics with the lowest performance. The low scores on these topics indicate that pre-service teachers had difficulty applying formulas, understanding concepts, and solving problems that were more procedural or required precise calculations.

Of all the topics, arithmetic series and fractions are the two materials with the lowest average and lowest scores, so it can be concluded that these two areas need the most strengthening in the learning process of pre-service teachers.

C. Analysis of Strategies and Difficulties of Pre-service Teachers

The section presents the results of an analysis of the thinking strategies and difficulties pre-service teachers experience when solving basic mathematical concept problems. This analysis focuses on pre-service teachers' thought processes while working on each problem item and on their reflections on the solution steps used. To explore the difficulties faced by pre-service teachers, the researchers analyzed the answers to the material with the lowest average scores: arithmetic series and fractions, with averages of 2.48 and 2.60, respectively.

An in-depth analysis of responses from six pre-service teachers, each representing the high, medium, and low-scoring categories, focused on the material with the lowest average scores: arithmetic series and fractions. This approach was used to illustrate variations in participants' understanding of the most challenging material and to identify differences in error patterns across categories. The following discussion summarizes the differences in the characteristics of the six participants' responses based on their performance categories in answering questions. The questions on arithmetic series and fractions are shown in Table 7.

Table 7. Arithmetic series and fraction questions

No.	Item	Questions
1	Arithmetic Series	$20+25+30+\dots+1000+12+16+20+\dots+200 = \dots$
2	Fractions	Explain mathematically and realistically the results of dividing the following two numbers: A. $\frac{0}{2}$ B. $\frac{0}{0}$

D. Case of Pre-service Teachers in the Higher-performing Participants

The pre-service teachers with high-performing participants were represented by pre-service teacher 1 (PST 1) with a total score of 80 and pre-service teacher 2 (PST 2) with a total score of 76. In solving arithmetic series and fraction problems, PST 1 scored 1 and 7, while PST 2 scored 5 and 5. Table 7 below describes the distribution of high-performing participants scores. The distribution of high-performing participants scores is shown in Table 8.

Table 8. Distribution of high-performing participants values

Item	PST 1	PST 2
Venn Diagrams	10	10
Sets	10	2
Algebra	10	10
Linear Inequalities in One Variable	9	10
Quadratic Equations	5	2
Fractions	7	5
Arithmetic Sequences	10	10
Arithmetic Series	1	5
Combinations	9	10
Permutations	9	9
Total Score	80	76

The analysis of the way of thinking of PST 1 and PST 2 in solving arithmetic series problems is shown in Table 9.

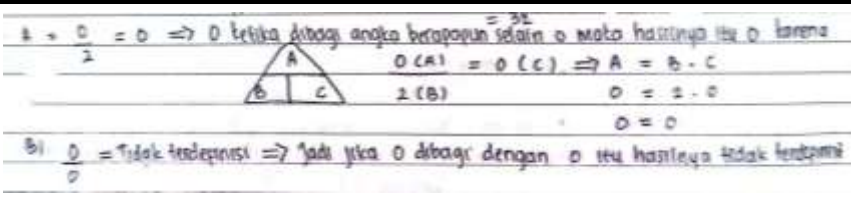
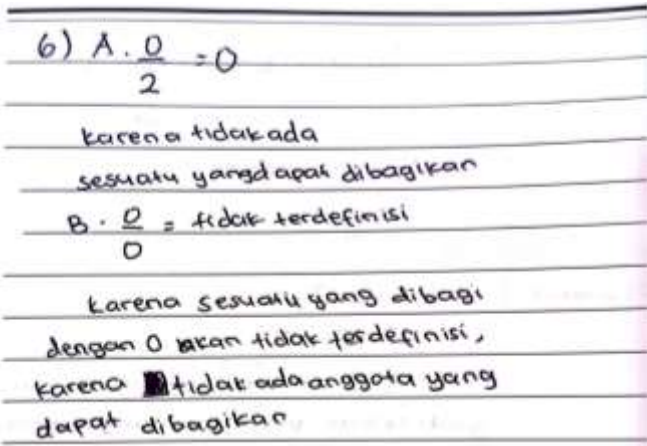
Table 9. Distribution of high-performing participants values

The result of PST 1	
Researcher	"How did you recognize that these numbers are an arithmetic sequence? What steps did you take to solve this problem?"
PST 1	"I recognize that these numbers have a pattern, from 20 to 25 and so on, and the same goes for 12 to 16, and so on. I usually work by grouping the number patterns first. However, in this problem, there are two different patterns, making it difficult for me to solve."
The result of PST 2	
Researcher	"What steps did you take to work on these two different patterns? Why did you choose those steps?"
PST 2	"I divided it into two groups, the first is the number group $20+25+30+\dots+1000$ with a difference of 5. At the same time, the second group is the number group $12+16+20+\dots+200$, with a difference of 4. The reason I chose this step is that it is clear that the difference between the first number group and the second number group is different, so I grouped them first and then entered them into the formula."

Based on the analysis of answers and interviews with PST 1 and PST 2, both already know how to solve arithmetic series involving groups of numbers with different differences. However, in their solutions, both PST 1 and PST 2 are not yet accurate. PST 1 found it difficult to solve two groups of numbers with different differences, while PST 2 is less precise at solving arithmetic series that have been grouped. According to Polya's problem-solving strategy for solving arithmetic series material, PST 1 and PST 2 apply the understanding of the problem and can identify information in the situation. However, PST 1 has not been able to devise a plan because it is confused about how to solve the problem and has not been able to carry it out or looking back. Meanwhile, PST 2 can apply by devising a plan, as it has used the formula according to the problem, but has not yet reached the stage of carrying out the plan and looking

back. Table 10 illustrates the results of the analysis of PST 1 and PST 2's answers to solving fraction problems.

Table 10. The thinking process of PST 1 and PST 2 in solving fractions

The result of PST 1	
Researcher	"What simple way did you use to solve this fraction problem mathematically and realistically?"
PST 1	"In solving this problem, I used a triangle consisting of the top side A, the left side B, and the right side C, which is commonly used in formulas for finding speed, distance, and time. I also applied it to solving this fraction problem. The number 0 is A, the number 2 is B, and C is a question mark. If C is the number 0, then in line with the result of 2 (B) multiplied by 0 (C), the result is 0 (A). If 0 (A) is reversed and divided by 2 (B), the result is 0."
The result of PST 2	
Researcher	"What steps did you take to solve the division of 0 by 2 and 0 by zero mathematically and realistically?"
PST 2	"My concept from the beginning was that when 0 is divided by something, the result will always be 0. Because if we compare it to everyday life, when we want to share with others, there is nothing to share or 0. Meanwhile, dividing 0 by 0 is undefined. I feel confused in making an analogy of the number 0 divided by a number."

Based on the answers and interviews with PST 1 and PST 2, who represent the group with high scores in explaining mathematically and realistically, PST 1 can answer the problem of 0 divided by 2 mathematically, but has not responded to the realistic form. PST 1 also has not provided a mathematical explanation for why the result 0:0 is undefined. Meanwhile, PST 2 can realistically answer both problems 0:2 and 0:0, but has not been able to explain them mathematically. According to Polya's problem-solving strategy for solving fraction material, PST 1 and PST 2 can both apply the steps of understanding the problem, devising a plan, and carrying out the plan. However, PST 1 and PST 2 have not reached the looking back stage because they have not included realistic examples.

E. Case of Pre-service Teachers in the Middle-performing Participants

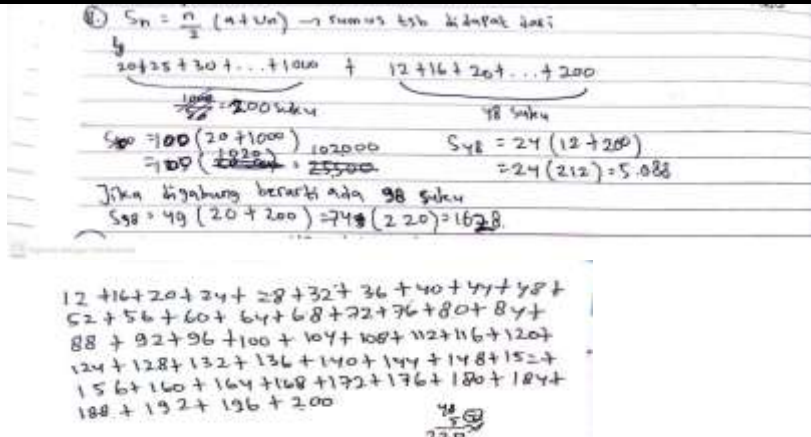
Pre-service teacher 3 (PST 3) and Pre-service teacher 4 (PST 4) are pre-service teachers in the middle-performing participants, with total scores of 67 and 66, respectively. In solving arithmetic series and fraction problems, PST 3 scored 2 and 5, while PST 4 scored 2 and 5. The following is Table 11, which describes the distribution of scores in the middle-performing participants.

Table 11. Distribution of values in the middle-performing participants

Item	PST 3	PST 4
Venn Diagrams	10	10
Sets	10	10
Algebra	3	8
Linear Inequalities in One Variable	10	10
Quadratic Equations	5	7
Fractions	5	5
Arithmetic Sequences	10	10
Arithmetic Series	2	2
Combinations	10	2
Permutations	2	2
Total Score	67	66

Furthermore, the analysis of PST 3 and PST 4 answers, as representatives of the middle-score group, in completing an arithmetic series, is shown in Table 12.

Table 12. The thinking process of PST 3 and PST 4 in solving arithmetic series

The result of PST 3	
Researcher	"Explain how you solve this arithmetic series problem?"
PST 3	"Actually, I was initially confused whether this arithmetic sequence was really divided into 2 different groups. After I checked by writing the addition one by one, I concluded that the addition was divided into 2 groups: the first addition with the number 5, which repeats like 20-25, and so on, until 1000. Then, for the numbers 12 to 16, the addition of 4 repeats until the number 200. After that, I just looked for (Sn) from each group."

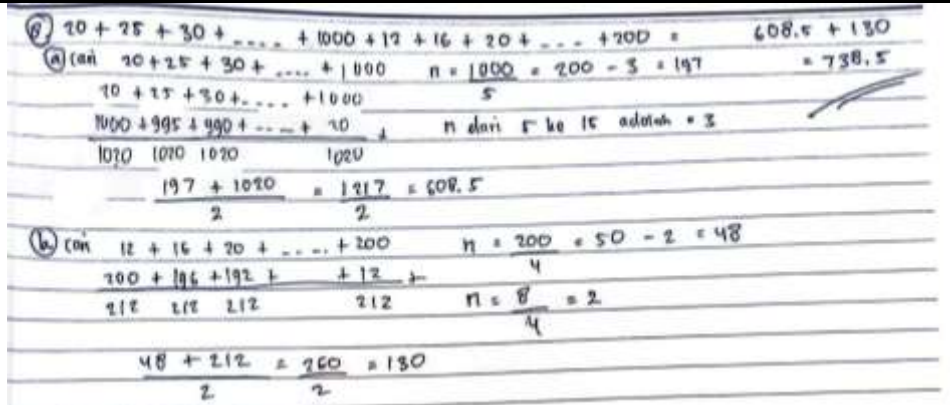
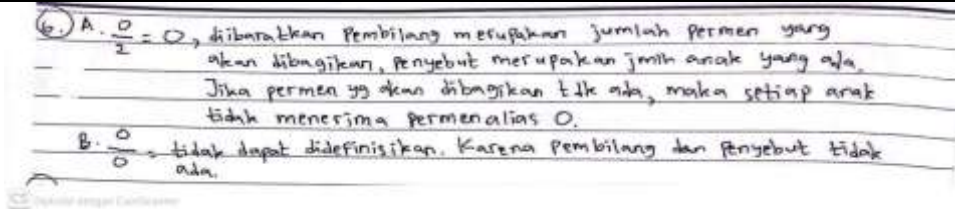
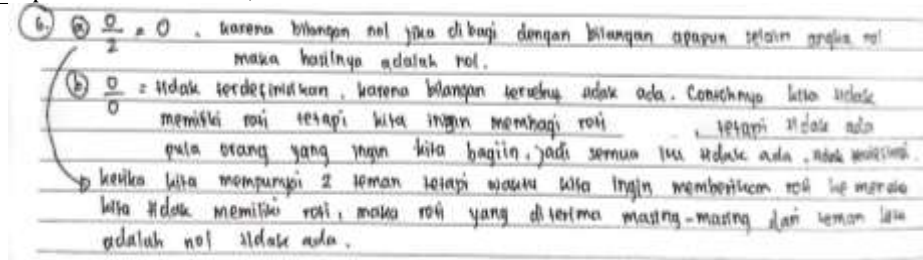
The result of PST 4	
Researcher PST 4	"What are your steps in solving this arithmetic series?" "After I saw the number pattern in the problem, I concluded that there were two calculations, then I broke it down into two groups of numbers. First, I worked on the $20+25+\dots+1000$ part, then I found the $12+16+\dots+200$ part. After I knew both, I added their results. However, I was doubtful whether my results were correct or not."

Table 13. The thinking process of PST 3 and PST 4 in solving fractions

The result of PST 3	
Researcher	"What is your view on solving mathematically and realistically the fraction problems $0:2$ and $0:0$?"
PST 3	"I imagine the denominator is the number of people, and the numerator is the number of items to be shared. For example, $0:2$ is the number of books to be shared between 2 children. The result must be 0 because there are no books to share among the children. Whereas $0:0$ means there are no items to be shared and no people to receive them, so it is called undefined."
The result of PST 4	
Researcher PST 4	"How do you explain $0:2$ and $0:0$ mathematically and realistically?" "Based on my experience, when calculating with a calculator, the number 0 divided by any number will definitely result in 0. However, when 0 is divided by any number, the result is undefined. For a simple way to solve $0:2$, I liken it to having 2 friends to give bread to, but we do not have any bread. So, the bread received from each friend is 0. For the problem $0:0$, the result is undefined because there is no item to share and no one to give it to."

Based on the analysis of answers and interviews with PST 3 and PST 4 regarding the solution of fractions, they can both realistically describe $0:2$ and $0:0$. PST 4 tends to use answers based on dividing 0 on a calculator. However, neither of them has been able to explain mathematically why $0:0$ is undefined, and $0:2$ is 2. According to Polya's problem-solving

strategy for solving fraction material, PST 3 and PST 4 can both apply understanding problems and devising a plan, because they can realistically state fractions. However, neither has reached the stage of carrying out the plan and looking back, because they have not been able to provide a mathematical explanation; they have only listed the answers.

F. Case of Pre-service Teachers in the Lower-performing Participants

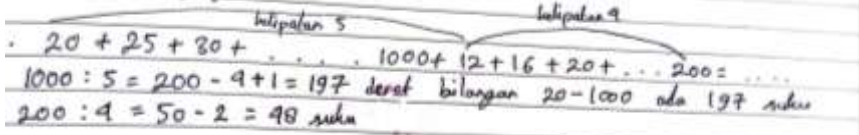
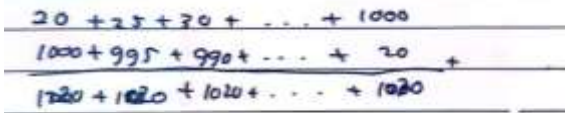
Based on the test results, pre-service teachers with lower-performing participants are represented by pre-service teacher 5 (PST 5) with a total score of 42 and pre-service teacher 6 (PST 6) with a total score of 27. In solving arithmetic series and fraction problems, PST 5 and PST 6 scored 2. Table 14 below describes the distribution of the lower-performing participants.

Table 14. The thinking process of PST 5 and PST 6 in solving fractions

Item	PST 5	PST 6
Venn Diagrams	9	2
Sets	10	10
Algebra	8	1
Linear Inequalities in One Variable	5	2
Quadratic Equations	2	2
Fractions	2	2
Arithmetic Sequences	2	2
Arithmetic Series	2	2
Combinations	0	2
Permutations	2	2
Total Score	42	27

To deepen the analysis of thinking on PST 5 and PST 6 as representatives of the lower-performing participants in solving arithmetic series problems, see Table 15.

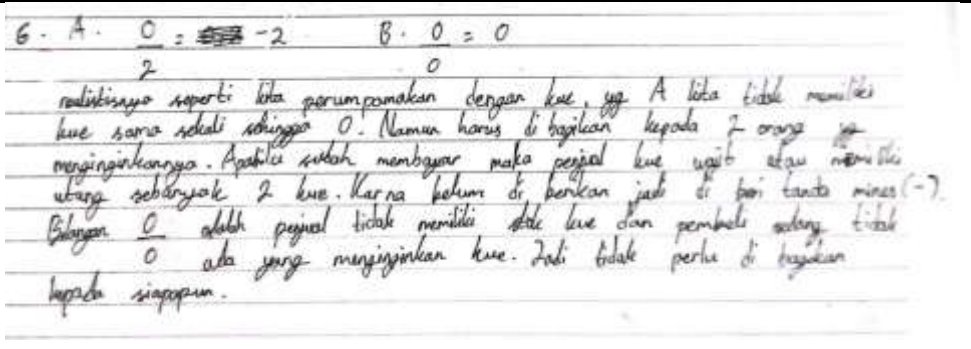
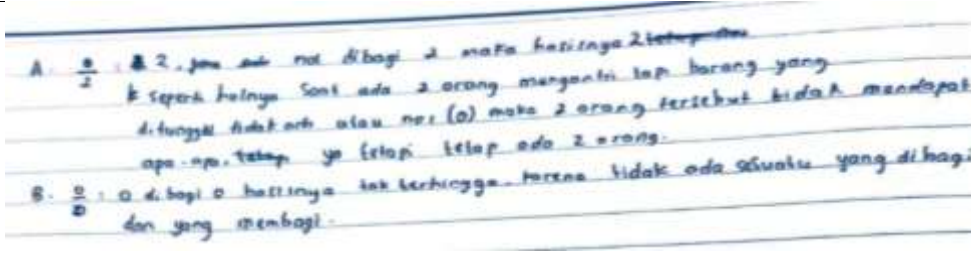
Table 15. The thinking process of PST 5 and PST 6 in solving arithmetic series

The result of PST 5	
Researcher	"How to solve an arithmetic series with this different pattern?"
PST 5	"I first grouped the number sequences according to their patterns. Since I often forget the formulas, I tend to use the manual method. I found it difficult because there were different number patterns within this arithmetic sequence. I was not sure whether to start with 0 or start with multiples of 5 and 4 in the first term."
The result of PST 6	
Researcher	"What are the steps in solving this arithmetic series problem?"
PST 6	"In solving this problem, I think I should use the formula directly because it will save time, but because I forgot the steps of the formula, I could not solve it properly."

Based on the analysis of answers and interviews with PST 5 and PST 6, both have different ways of thinking when solving this arithmetic series problem. PST 5 tends to use manual methods without formulas, while PST 6 does not; both are equally inaccurate in solving this

arithmetic series problem. According to Polya's problem-solving strategy for solving arithmetic series problems, PST 5 and PST 6 can both apply their understanding of the problem. However, neither has reached the stages of devising a plan, carrying it out, and reflecting on it. Therefore, neither has been able to solve the problem of an arithmetic series. To analyze the answers of PST 5 and PST 6 in solving fraction problems, see Table 16.

Table 16. The thinking process of PST 5 and PST 6 in solving fractions

The result of PST 5	 6. A. $\frac{0}{2} = -2$ B. $\frac{0}{0} = 0$ realistisnya seperti kita perumpamaan dengan kue, yg A kita tidak memiliki kue sama sekali sehingga 0. Namun harus di bagikan kepada 2 orang yg menginginkan. Apabila sudah membayar maka penjual kue wajib atau memiliki utang sebanyak 2 kue. Karena belum di berikan jadi di beri tanda minus (-). Bilangan 0 adalah penjual tidak memiliki kue dan pembeli sedang tidak ingin kue. Jadi tidak perlu di bagikan kepada siapapun.
Researcher	"What do you think about solving the division mathematically and realistically?"
PST 5	"The first thing that came to my mind regarding the division of $0:2$ and $0:0$ was that the numerator is the amount of something we have to share, and the amount is nothing at all because the number is 0. When 0 is divided by 0 the result is 0. I felt that $0:2$ was a difficult problem, so I mistakenly thought of it as subtraction, because I thought that 0 was something that had not been shared with 2 people, so I used the minus sign (-2)."
The result of PST 6	 A. $\frac{0}{2} = 2$ B. $\frac{0}{0} = \infty$ 0 : 2, jika ada kue dibagi 2 maka hasilnya 2. Jika ada 2 orang, masing-masing 1 kue. Barang yang ditunggu tidak ada atau nol (0) maka 2 orang tersebut tidak mendapatkan apa-apa, tetapi ya tetap tetap ada 2 orang. B. $\frac{0}{0} = 0$ dibagi 0 hasilnya tak terhingga, karena tidak ada sesuatu yang dibagi dan yang membagi.
Researcher	"How do you solve $0:2$ and $0:0$?"
PST 6	"To solve this problem, I used an analogy. For example, if we have $0:2$, we have 2 friends but no items to share. However, I still have 2 friends, so the result of $0:2$ is 2. Then, for $0:0$, the result is infinity, because there are no friends to share anything with and no items to share. I have somewhat forgotten the concept of division."

Based on the analysis of answers and interviews with PST 5 and PST 6 regarding solving fraction problems, it can be concluded that both have the same incorrect concept regarding the analogy of the results of $0:2$ and $0:0$. This could be because the pre-service teachers forgot the idea of division by 0, so they were less precise in answering it. According to Polya's problem-solving strategy for solving fraction material, PST 5 and PST 6 have not yet implemented the stages of understanding the problem, devising a plan, carrying out the plan, and looking back. Both apply the concept and answer incorrectly.

G. Conceptual Difficulties and Error Patterns of Pre-service Teachers

The findings of this study indicate that the mastery of elementary school pre-service teachers on basic mathematical concepts is still very varied, which is reflected in the distribution of scores on ten basic mathematical topics, including Venn diagrams, sets, algebra, linear inequalities with one variable, quadratic equations, fractions, arithmetic sequences, arithmetic series, combinations, and permutations. Quantitative results show that the two topics most difficult for pre-service teachers are arithmetic series and fractions. This is consistent with previous research, which confirms that fractions and series operations are the areas that cause the most misconceptions in pre-service teachers.

In the context of qualitative analysis, it is used to explain further the error patterns observed in the test results. The results of interviews with six PSTs with higher-performing participants, middle-performing participants, lower-performing participants revealed that the difficulties pre-service teachers faced varied according to their level of mastery. Participants with higher-performing, namely PST 1 and PST 2, demonstrated the ability to recognize number patterns and provide more explicit conceptual reasoning. However, they still showed inaccuracies in their solution steps, especially in arithmetic series problems with two different patterns. This indicates that although their procedural mastery is quite good, their conceptual understanding is not yet fully stable, as Jiang et al. (2022) argued that pre-service teachers often operate in a "transitional understanding" when building mathematical concepts.

Participants in the middle-performing participants demonstrated partial understanding, understanding the general idea but frequently making procedural errors or being unsure about the final step. When solving arithmetic sequences, PST 3 and PST 4 were able to distinguish between two groups of patterns. However, they were unable to accurately determine the final result or ensure the correctness of their calculations. This finding echoes those of Márquez et al. (2021), who found that many pre-service teachers understood the concept but were unable to apply mathematical reasoning consistently. Participants with lower-performing participants demonstrated fundamental misconceptions in both fraction operations and the interpretation of arithmetic sequences. PST 5 and PST 6, for example, misunderstood the meaning of the divisions $0:2$ and $0:0$, and frequently combined division with subtraction or similes, which are not mathematically appropriate. These findings support Tfali's (2023) study, which found that pre-service teachers often misunderstand the concept of fractions and struggle to explain it mathematically.

When comparing higher-performing, middle-performing, and lower-performing participants, the pattern of findings suggests that the primary difference lies not only in the final results but also in the quality of mathematical reasoning. Higher-performing participants used more systematic strategies and constructed better internal representations of the

problems they encountered. Middle-performing participants recognized the problem structure but were inconsistent in applying the correct algorithm. Meanwhile, lower-performing participants tended to use informal metaphors that were not mathematically relevant. These findings support Nunes et al.'s (2011) view that understanding basic mathematical concepts depends heavily on teachers' ability to construct correct mental representations of the concepts they teach.

From a higher-order thinking skills perspective, this study's results also show that most pre-service teachers have not demonstrated strong analytical skills in solving mathematical problems. In fact, critical thinking skills are an important foundation for pre-service teachers to develop meaningful mathematics learning (Serin et al., 2018; Abalı Öztürk, 2022). The findings of this study indicate that pre-service teachers still need more intensive training in developing mathematical thinking (Çelik & Özdemir, 2020) especially in breaking down problems into structured steps and choosing appropriate strategies.

H. Comparison of Problem-Solving Strategies Across Categories of Pre-service Teachers

Comparing problem-solving strategies across higher-, middle-, and lower-performing participants provides crucial insights into the quality of reasoning underlying participants' performance, particularly when viewed through Polya's framework. Key failures often occurred at Stage 1 (understanding the problem) among middle- and lower-performing participants, who struggled to identify the known and requested information, leading to misinterpretations of problem scenarios. This difficulty continued into Stage 2 (devising a plan), where they struggled to select or formulate relevant strategies, often confusing formulas or using intuitive approaches that were inconsistent with mathematical rules. In contrast, higher-performing participants demonstrated strengths in the first two Polya stages by constructing a correct mental representation of the problem and systematically planning the solution procedure. However, their understanding appeared to remain transitional, as they could correctly execute procedures at Stage 3 (carrying out the plan), but their flexibility and depth of conceptual explanation remained limited when faced with a variety of problems. This suggests that the primary differences in performance lie not merely in procedural knowledge but in the quality of mathematical reasoning and the ability to connect concepts necessary to construct a coherent strategy early in the problem-solving process.

I. Demands for Higher Order Thinking Skills in Understanding Mathematical Concepts

The difficulties encountered in this study directly reflect the demands on Higher Order Thinking Skills (HOTS) essential for mastery of mathematics, particularly those related to Polya's stages 3 and 4. Analysis shows that in stage 3 (carrying out the plan), errors made by the medium and low groups went beyond missed calculations to include inconsistent

procedural execution due to conceptual instability, such as mid-strategy changes. The most significant error observed in the results was that nearly all participants, including the high-performing participants, demonstrated weaknesses in stage 4 (looking back). Participants rarely revisited the logical and mathematical validity of their answers or attempted alternative verification methods. This lack of checking is a strong indicator that participants lack the metacognitive and self-correction habits that are key components of HOTS. For pre-service teachers, this failure to develop HOTS is crucial, as they must be able not only to solve problems but also to model these thinking and reasoning processes to elementary school students. Therefore, this discussion emphasizes the need to shift the focus of pre-service teacher education away from numerical accuracy and towards strengthening abstract reasoning and analytical thinking skills as a foundation for accurate conceptual understanding.

J. Pedagogical Implications and Direction of Conceptual Strengthening

As a synthesis of the discussions on conceptual difficulties, strategic patterns, and HOTS demands, the findings of this study have significant pedagogical implications for preservice teacher education. To address mastery deficits in fractions and arithmetic series, educational programs should prioritize instructional methods that explicitly teach Polya's problem-solving framework and foster deep conceptual reinforcement. A recommended approach in this regard is the use of a Problem-Based Learning (PBL) approach designed to systematically train students in stages 1 (understanding the problem) and 2 (devising a plan) of the problem, thereby minimizing initial misconceptions. Furthermore, instruction should provide sufficient space for conceptual discussions and reflective activities that specifically emphasize stage 4 (looking back), encouraging students to develop habits of self-correction and metacognition. Practically, this means a paradigm shift from instruction focused on procedures to instruction focused on reasoning and understanding. Thus, a key recommendation of this study is the need for curricular interventions to equip pre-service teachers with solid conceptual understanding and strong HOTS, ensuring they are prepared to become educators capable of fostering practical mathematical thinking in the next generation of pre-service teachers.

Conclusion

This study concludes that mastering basic mathematical concepts, particularly arithmetic series and fractions, remains a critical challenge for pre-service teachers. The results indicate that the primary difficulty for students lies not solely in arithmetic errors, but rather in fundamental misconceptions and conceptual instability, as evidenced by inconsistent error patterns and the use of informal reasoning.

A thorough analysis of problem-solving strategies using Polya's framework revealed significant weaknesses in stages 1 (understanding the problem) and 4 (looking back). Middle- and lower-performing participants often failed to interpret problem information correctly,

leading to incorrect solution planning at stage 2 (devising a plan). Crucially, participants across all performance levels demonstrated severe weaknesses in self-correction and metacognition at stage 4, indicating that their mathematical reasoning and higher-order thinking skills (HOTS) were not yet optimally developed.

Therefore, it is concluded that pre-service teacher education programs need to revise their curricula by emphasizing in-depth conceptual instruction and systematic problem-solving training, utilizing the Polya framework. A primary focus should be on developing analytical and reflective skills, as well as on using multiple representations, to ensure pre-service teachers have a solid understanding of mathematics and can model critical thinking processes for their future students.

Conflict of Interest

The authors declare that there is no conflict of interest.

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