



Students' Use and Coordination of Symbolic and Number-Line Representations in Fraction Comparison: A Qualitative Multiple-Case Study

¹Miftahul Jannah, ¹Rustanto Rahardi*, ¹Dahliatul Hasanah

Corresponding Author: *rustanto.rahardi@fmipa.um.ac.id

¹ Universitas Negeri Malang, Malang, Indonesia

ARTICLE INFO

Article history

Received 06 June 2026

Revised 03 September 2026

Accepted 05 September 2026

ABSTRACT

Using more than one representation in fraction comparison does not, by itself, indicate that students have coordinated the mathematical relationships across representations. This study aimed to analyze the use and functions of symbolic and number-line representations, shifts and coordination across representations, and the final basis on which students accepted their answers as correct when comparing fractions. A qualitative multiple-case study design was employed. Twenty-four seventh-grade students completed an open-ended task comparing $5/12$ and $4/9$. Based on variation in written representational patterns, four focal cases (S7, S11, S17, and S24) were purposively selected and participated in two rounds of task-based interviews. Data were analyzed using a hybrid deductive-inductive thematic analysis integrating written work, interview data, and cross-case comparison. The findings showed that the number line served different functions across cases, including initial solution search, visualization, explanation, and verification, whereas symbolic procedures were used to obtain, clarify, and justify results. Conversion, cross-representational consistency, and conceptual coordination also showed distinct patterns: evidence of coordination was partial for S7, could not be established for S11, was stronger within the task context for S17, and could not be assessed across representations for S24. For S7, S11, and S17, symbolic procedures carried greater justificatory weight than the number line, whereas for S24, the symbolic procedure was the only observable procedural basis. These findings highlight the importance of distinguishing representational form from function, conversion and cross-representational consistency from conceptual coordination, and

representational function from justificatory function when analyzing students' reasoning in fraction-comparison tasks.

Keywords

Conceptual Coordination
Final Warrant for Correctness
Fraction Comparison
Mathematical Representations
Number Line

This is an open-access article under the [CC-BY-SA](#) license.



Introduction

Fractions are a fundamental concept in mathematics education because they are closely related to the development of understanding of ratios, proportions, and more advanced mathematical concepts. Understanding fractions is also associated with students' success in learning mathematics at higher levels [1]–[5]. Therefore, competence with fractions cannot be demonstrated solely through the ability to perform computational procedures; it also requires understanding fractions as numbers with particular values or magnitudes.

Despite their importance, fractions remain challenging for students across different educational levels [6]–[8]. One such difficulty arises when students compare two fractions, particularly those with different denominators. In these situations, students may treat the numerator and denominator as two separate whole numbers and rely on whole-number intuitions to determine which fraction is larger. This tendency, commonly referred to as whole-number bias, may lead students to overlook the proportional relationship among the numerator, denominator, and fraction magnitude [9]–[10]. Thus, fraction-comparison tasks require not only obtaining the correct comparison but also making visible how students interpret the values of the two fractions and justify the relationship between them.

Different mathematical representations can support reasoning in fraction comparison. Symbolic representations, for example through the construction of equivalent fractions with common denominators, provide a systematic means of comparing fraction values. Visual representations such as number lines, by contrast, allow fractions to be viewed as numbers occupying particular positions according to their magnitudes. Number-line representations have been reported to support students' understanding of fraction value and magnitude [11]–[12]. However, this potential depends on how students interpret the partitions, positions, intervals, and values represented on the number line. Patahuddin et al. [13], for example, showed that the conceptual opportunities afforded by number lines are not always taken up by students in ways that correspond to the intended mathematical meanings.

This suggests that the presence of multiple representations within a solution does not, by itself, provide evidence that students understand the conceptual relationships among those

representations. Students may use both symbolic and visual representations and obtain consistent results without necessarily being able to explain how the two representations express the same quantitative relationship [14]. It is therefore important to distinguish movement or conversion across representations from conceptual coordination across representations. Conversion refers to a shift from one representational form or registers to another, whereas coordination requires evidence that students can relate the meanings or invariant quantitative relationships preserved across the representations they use. Accordingly, a correct answer or the use of more than one representation cannot automatically be treated as evidence of conceptual understanding.

These distinctions can be understood through several perspectives on mathematical representation. Goldin [15] positions representational systems as an important component of mathematical learning and problem solving and suggests that strategies, heuristics, external representations, and monitoring activities can be examined through observable mathematical behavior. This perspective also calls for caution when inferring internal structures or processes from external evidence. Duval [16] emphasizes that representational activity involves not only processing within a register but also conversion across registers and coordination of the mathematical relationships preserved between them. Meanwhile, Ainsworth's framework of multiple external representations [17] suggests that multiple representations may serve complementary functions, constrain interpretation, or support the construction of deeper understanding, although these benefits depend on how learners understand and connect the available representations. Drawing on these three perspectives, the present study examines not only which representations appear in students' solutions but also the functions that can be traced to their use and the relationships students establish across representations.

In fraction comparison, this issue is particularly relevant because the same student may use different representations for different purposes. A number line, for example, may be used to form an initial estimate, visualize the positions of fractions, explain relationships between values, or check a result obtained through another method. Symbolic procedures, in contrast, may be used both to obtain a comparison result and to provide final justification for an answer. Thus, the use of multiple representations does not necessarily mean that all representations serve the same function or carry the same justificatory weight in students' reasoning. The analytic focus therefore needs to move beyond asking simply, "Which representations do students use?" toward examining what each representation is used for, how students move between and connect representations, and which representation or procedure ultimately serves as the basis for accepting an answer as correct.

Previous research has demonstrated the potential of number lines for representing fraction magnitude as well as students' difficulties in interpreting and connecting symbolic and visual representations [11]–[14]. Other studies have also highlighted the need to distinguish procedural success from conceptual understanding in fractions and have shown that particular procedures may be used predominantly in fraction-comparison tasks [18]–[20]. However, within the literature informing the present study, these issues have largely been examined from separate perspectives. Limited attention has been given to analyses that, within the same cases and task, simultaneously distinguish (1) the functions served by symbolic and number-line representations, (2) conversion across registers, consistency of outcomes, and conceptual coordination, and (3) the representation or procedure that students ultimately treat as the basis for accepting an answer as correct. The present study addresses this analytic space through an in-depth examination of four cases within a single fraction-comparison task. The analysis is deliberately bounded to evidence observable in students' written work and task-based interviews; it is not intended to infer students' mental processes in general or to explain the instructional causes of the representational strategies they use.

One aspect examined in this study is the final basis for accepting an answer as correct, referred to here as the final warrant for correctness. In this study, the term refers to the representation or procedure that a student treats as the determining basis for accepting an answer as correct within the given comparison task. This construct is identified primarily through evidence that a student assigns greater justificatory weight to a particular representation or procedure, for example by describing it as more convincing, more certain, or more trustworthy when deciding whether an answer is correct. Final warrant for correctness is not equated with frequency of strategy use, verification function, visual preference, procedural accuracy, or conceptual understanding. A student may therefore use both a number line and a symbolic procedure while still treating only one of them as the final basis for accepting the answer.

Based on this analytic gap, the study is framed as a qualitative multiple-case investigation of students' observable reasoning in fraction comparison, drawing on written work and task-based interviews. It does not seek to establish students' cognitive processes in general or to explain causal relationships between particular instructional practices and the strategies students use. Instead, the study examines in depth how four selected cases used symbolic and number-line representations when comparing $\frac{5}{12}$ and $\frac{4}{9}$, the functions served by each representation, how students moved between and coordinated representations, and which representation or procedure served as their final basis for justification.

Specifically, the study addresses the following research questions:

1. How did the four focal students use symbolic and number-line representations when comparing $5/12$ and $4/9$, and what functions did each representation serve in their solutions?
2. How did these students move between symbolic and number-line representations, and what evidence indicated the extent of coordination between these representations when they explained, checked, and justified their solutions?
3. Which representation or procedure could be identified as the final basis for justification in each case, and what observable evidence supported or limited that identification?

Material and Methods

A. Research Design and Context

This study employed a qualitative approach using a multiple-case study design to examine students' use and coordination of mathematical representations in a fraction-comparison task. The analysis focused on reasoning that could be observed through students' written work and verbal explanations rather than on general inferences about internal cognitive processes.

The study was conducted in January 2026 at MTs Muhammadiyah Balassuka, Gowa Regency, Indonesia, during the 2025/2026 academic year. The initial stage involved 24 seventh-grade students aged 13–14 years from one class. All students who were present during data collection completed the same written task. According to the mathematics teacher, the students had previously been introduced to several strategies for comparing fractions, including finding common denominators, cross multiplication, decimal conversion, and number lines. Because the study did not directly observe prior classroom instruction, this information was used only to describe the students' instructional background and not to explain the causes of the representational strategies they used.

All 24 written responses were examined to map initial variation in strategies and representational use. The in-depth analysis reported in this study then focused on four selected cases: S7, S11, S17, and S24.

B. Fraction-Comparison Task and Focal-Case Selection

All students completed one open-ended task requiring them to compare $5/12$ and $4/9$, determine which fraction was larger, explain their reasoning and solution steps, and use a drawing or number line if needed. The complete task wording is provided in Supplementary Appendix A.

The task was developed after the researcher reviewed examples of fraction-comparison tasks in previous studies and refined it through repeated consultation with the research supervisor. The instrument did not undergo a formal expert-validation procedure or a separate pilot test; therefore, its development is described as an iterative review and refinement with the research supervisor.

The pair $5/12$ and $4/9$ was selected because the two fractions have relatively close magnitudes: $5/12 = 15/36$, $4/9 = 16/36$, with a difference of $1/36$. This characteristic allowed for the use of multiple solution strategies while providing a context in which to examine how students compared and justified two fractions with relatively close values.

Students were given 30 minutes to complete the task and were allowed to use a ruler. The researcher did not provide solution hints or assistance during task completion, and the number line was mentioned only as an optional representation in the instructions. Students were instructed to work individually; however, some students engaged in brief discussions about the problem, strategies, or answers during the task. This condition was considered part of the data-collection context and a limitation on interpretation.

The four focal cases were purposively selected based on variation in the representational patterns observed in the written responses rather than on categories of academic ability. After potential cases had been identified, the mathematics teacher provided additional information about the students' ability to communicate their solution reasoning. The teacher did not view the written work used in the selection process; thus, this information served only as a secondary consideration.

Table 1. Initial Representational Patterns of the Four Focal Cases

Case	Initial representational pattern
S7	Common-denominator procedure followed by a number line
S11	Number line as the initial strategy, followed by a symbolic procedure
S17	Common-denominator procedure followed by a number line with a more explicit quantitative explanation
S24	Solution using symbolic procedures only

This selection produced contrasting cases within the same classroom dataset, allowing similarities and differences in representational use to be examined in depth.

C. Task-Based Interviews

The four focal students participated individually in two rounds of interviews conducted via Google Meet. The first interview took place several days after the written task, whereas the second was conducted approximately one week later. The total duration of the two interviews was approximately 30 minutes per student.

During the interviews, students were shown their own written work and asked to explain their solution steps, reasons for selecting particular strategies, and the functions of the representations they had used. The interviews covered similar question domains across participants, while the sequence, wording, and follow-up probes were adapted to each student's written work and responses. The domains included the initial strategy, reasons for strategy selection, functions of symbolic procedures and number lines, relationships between

position and magnitude, answer verification, and the basis of justification considered more convincing.

The second interview served as a follow-up clarification interview. It was used to elaborate responses that remained unclear in the first interview, introduce additional questions following consultation with the research supervisor, and examine the consistency of students' explanations. The second interview was not a learning intervention and was not treated as temporal triangulation.

The interviews were conducted in informal Indonesian, with some naturally occurring Makassar-influenced conversational expressions. All interviews were audio-recorded and transcribed verbatim by the researcher. The transcripts were subsequently checked against the audio recordings. Student quotations included in the English-language manuscript were translated by the researcher and reviewed with the research supervisor, while the original-language versions were retained as the source material for analysis. Further details on the interview domains and representative prompts are provided in Supplementary Appendix B.

D. Data Analysis and Analytic Codebook

The data were analyzed using a hybrid deductive–inductive thematic analysis. Initial coding was theory-informed and guided primarily by perspectives on mathematical problem solving and representation from Polya [21], Goldin [15], and Duval [16]. The principles of thematic analysis proposed by Braun and Clarke [22] were applied flexibly to support repeated reading, coding, pattern comparison, and theme development rather than being presented as a six-phase procedure that had been followed mechanically from the outset.

In the first stage, all 24 written responses were examined to map variation in strategies and representational use and to support focal-case selection. After S7, S11, S17, and S24 had been selected and interview data were available, the written responses and both interview rounds for the four cases were analyzed in an integrated manner. Through repeated reading and comparison across the data, the initial categories were refined into a final codebook comprising eight analytic categories.

Following Duval [16], a transformation such as $5/12 = 15/36$ was treated as a treatment because it occurs within the same symbolic register, rather than as a conversion. Conversion was coded when a student moved between different representational registers, for example from symbolic fractions to a number line or vice versa. Cross-representational consistency (CR) was also distinguished from conceptual coordination (CC): two representations could produce the same final comparison without sufficient evidence that the student could explain the quantitative relationship connecting them.

Final warrant for correctness was not part of the initial coding categories. It emerged inductively from comparisons across the four cases when the analysis revealed differences

between the representations students used and the representation or procedure to which they assigned greater justificatory weight.

Table 2. Analytic Categories and Coding Foci in the Codebook

Code	Category	Analytic focus
IS	Initial Strategy	The first strategy or representation used or reported by the student
SF	Symbolic Function	The function served by symbolic procedures or representations
VF	Visual Function	The function of the number line in searching, visualizing, explaining, or verifying
CV	Conversion	Movement between different representational registers
CR	Cross-Representational Consistency	Product-level consistency between comparison relationships produced through two representational registers
CC	Conceptual Coordination	Evidence of quantitative relationships preserved across representations
MJ	Monitoring and Justification	Checking, correcting, evaluating, and providing reasons for a solution
FW	Final Warrant for Correctness	The representation or procedure treated as the most decisive basis for accepting an answer as correct

This category was therefore distinguished from frequency of strategy use, ease of use, representational preference, procedural accuracy, and conceptual understanding. Ainsworth [17] was used during the interpretive stage to understand variation in the functions of multiple representations, whereas Skemp [23] was used only in the Discussion as an interpretive lens.

After each case had been analyzed using IS, SF, VF, CV, CR, CC, MJ, and FW, the coding results were organized in a case-by-category matrix to compare code configurations, supporting evidence, and evidence that limited interpretation across the four cases. The analysis was not based on code frequency. Cross-case comparison was synthesized into three main patterns: (1) variation in the functions of symbolic and number-line representations; (2) distinctions among conversion, cross-representational consistency, and conceptual coordination; and (3) final warrant for correctness. Operational definitions, inclusion and exclusion criteria, and illustrative evidence are provided in Supplementary Appendix C, while examples showing the links among data, codes, and interpretations are provided in Supplementary Appendix D.

E. Analytic Trustworthiness

Analytic rigor was supported through several procedures. Interpretations were developed by comparing students' written work with interview data, using the second interview to clarify unclear responses and examine consistency in students' explanations, checking transcripts against the audio recordings, and discussing coding tables and selected interpretive decisions with the research supervisor.

Coding was conducted by the primary researcher without an independent second coder, inter-rater reliability analysis, or Cohen's kappa; therefore, the study does not claim inter-rater reliability. Instead, the traceability of analytic decisions was strengthened through an explicit codebook, a cross-case matrix, direct links between findings and written or verbal evidence,

and the retention of limiting evidence. Such evidence included S7's initial uncertainty, S11's unstable interpretation of the number line, and the inconsistency between S24's written justification and verbal explanation.

F. Data Availability Statement

The raw data, including audio recordings and complete interview transcripts, are not publicly available in order to protect participant confidentiality. Anonymized evidence supporting the analysis is presented in the manuscript and the Supplementary Materials.

Results

Analysis of the written work and two rounds of interviews from the four focal cases revealed variation in initial strategies, the functions of symbolic and number-line representations, shifts across representations, and the strength of evidence for conceptual coordination. Cross-case comparison also revealed patterns in the representations or procedures students treated as the basis for accepting their solutions as correct. Table 3 summarizes the main patterns across the four cases based on written work and interview explanations.

Table 3. Cross-Case Patterns in Representational Use and Coordination

Participant	Sequence of representational use	Representational functions	Evidence of conceptual coordination	Final basis for justification
S7	Symbolic → number line	Symbolic: obtaining and justifying the result; visual: visualization and verification	Partial	Symbolic procedure
S11	Number line → symbolic	Visual: initial solution search; symbolic: clarifying and confirming the result	Could not be established from the available evidence	Symbolic procedure
S17	Symbolic → number line	Symbolic: obtaining and justifying the result; visual: visualization, meaning-based explanation, and verification	Strong within the task context	Symbolic procedure
S24	Symbolic only	Symbolic: obtaining and justifying the result; visual: not used	Not assessable across representations	Not comparatively assessable; observed procedural basis: symbolic

Overall, S7, S17, and S24 identified the common-denominator procedure as their initial strategy, whereas S11 identified the number line as the initial strategy. S7 and S17 subsequently used visual representations after symbolic procedures, S11 moved from the number line to a symbolic procedure, and S24 remained within the symbolic register. However, differences in this sequence did not necessarily indicate equivalent representational functions or the same level of conceptual coordination.

A. Functions of Symbolic and Number-Line Representations

In S7's written work, $\frac{5}{12}$ and $\frac{4}{9}$ were first expressed as equivalent fractions with a common denominator, $\frac{15}{36}$ and $\frac{16}{36}$. After establishing that $\frac{15}{36} < \frac{16}{36}$, S7 represented the two fractions on a number line. Thus, the written work showed a symbolic procedure followed by a visual representation.

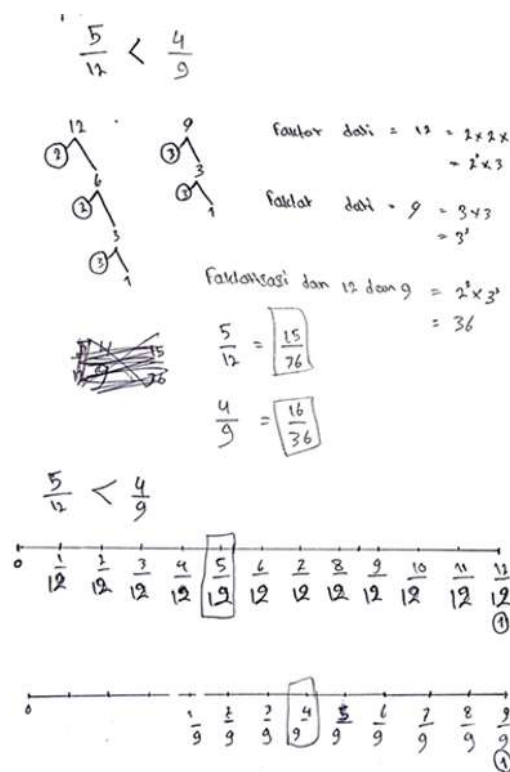


Fig. 1. S7's written work showing the common-denominator procedure followed by representations of $\frac{5}{12}$ and $\frac{4}{9}$ on a number line

This sequence was consistent with S7's interview explanations. In the first interview, S7 stated that the number line was used "to make sure again whether the answer was correct or not" (S7W14). In the second interview, S7 explained that the number line was used "to see the positions of the numbers visually" (S7W46). S7 also stated that the drawing helped with understanding the concept (S7W85). Taken together, the written work and interview data indicate that the number line primarily functioned to visualize fraction positions and verify a result that had previously been obtained symbolically. S7's statement that the drawing

supported conceptual understanding was treated as supporting evidence rather than as the primary function assigned to the number line in this case.

S11's written work showed a different pattern. The response sheet contained a number line, a common-denominator procedure, and several crossed-out symbolic steps. However, the chronological sequence of all these steps could not be determined solely from their spatial arrangement on the page.

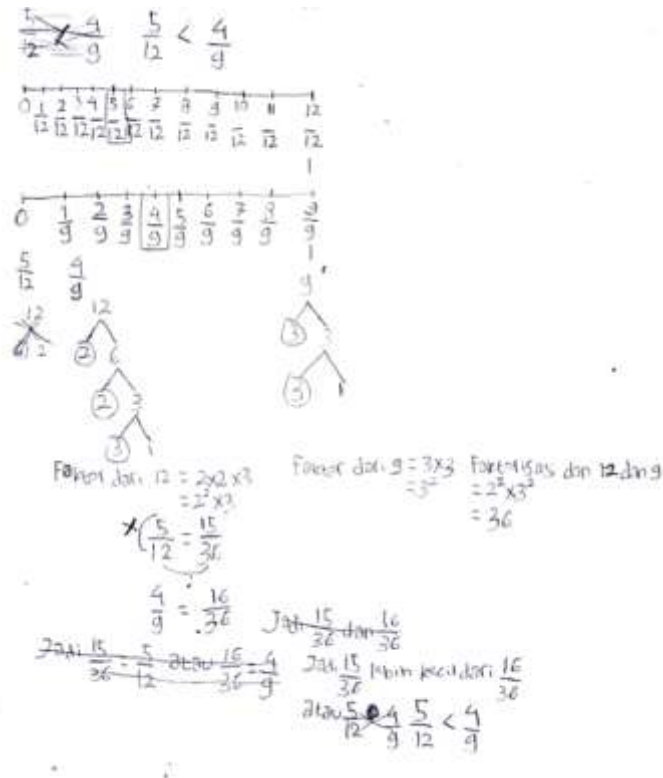


Fig. 2. S11's written work containing a number line, a common-denominator procedure, and corrected parts of the solution

The interview provided clearer evidence regarding the sequence of S11's strategies. When asked what first came to mind, S11 stated, "I tried to find the answer using a number line" (S11W22). When asked about the purpose of the number line, S11 responded, "To find the answer" (S11W28). The common-denominator procedure was then used to obtain a result that S11 considered clearer (S11W54). Based on the combined evidence, S11's pattern was categorized as visual-to-symbolic: the number line functioned as an initial means of searching for or determining the answer, whereas the symbolic procedure was used to clarify and confirm the result.

In S17's written work, the two fractions were first converted to equivalent fractions with a common denominator, $\frac{15}{36}$ and $\frac{16}{36}$, and were then represented on a number line. As with S7, S17's sequence therefore moved from symbolic to visual representation.

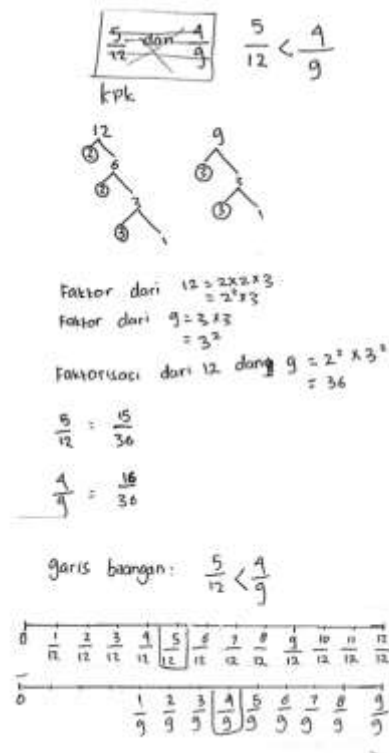


Fig. 3. S17's written work showing the common-denominator procedure and the representation of both fractions on a number line

The interview explanations indicated that the number line served a broader function for S17 than simple checking. S17 explained that the number line was used "to visualize the positions of the numbers and make sure that the calculation made logical sense" (S17W48). S17 also stated that the number line helped with understanding the concept and checking for possible calculation errors (S17W50). Based on this evidence, the number line in S17's solution functioned to visualize magnitude, support explanation of quantitative meaning, and verify the symbolic result.

S24 showed a different pattern from the other three cases. The written work contained only symbolic procedures involving the identification of a common denominator and the formation of the equivalent fractions $\frac{15}{36}$ and $\frac{16}{36}$, with no number line or other visual representation.

This pattern was consistent with S24's explanation that "the first thing is, we have to make the denominators the same and then compare" (S24W14). After the denominators had been made equal, S24 explained that the comparison was made by looking at the numerators (S24W42). Thus, the symbolic procedure functioned both to obtain and justify the result, whereas a visual function could not be analyzed because no visual representation was used.

Across cases, these findings show that the presence of a number line did not, by itself, determine a particular representational function. Within the same task, the number line served

as an initial solution-search tool for S11, as a means of visualization and verification for S7, and as a means of visualization, explanation, and verification for S17. S24 did not use a number line. Representational functions therefore need to be identified from how students use and explain a representation, rather than merely from its presence in written work.

~~$\frac{5}{12}$ dan $\frac{4}{9}$~~ $\frac{5}{12} < \frac{4}{9}$
 cara kerja. dengan menggunakan lipatan.
 12 = 24.
 12, 24, (30), 48 Faktorisasi dari 12 = 12×3
 9, 18, 27, (36) Faktorisasi dari 9 = 9×3

$$\frac{5 \times 3}{12 \times 3} = \frac{15}{36}$$

$$\frac{4 \times 4}{9 \times 4} = \frac{16}{36}$$
 Jadi $\frac{5}{12}$ lebih kecil dari $\frac{4}{9}$
 karena lebih kecil pembilangnya maka lebih besar hasilnya.

Fig. 4. S24's written work showing a solution based on the common-denominator procedure without the use of a visual representation

B. Conversion, Cross-Representational Consistency, and Conceptual Coordination

The use of more than one representational register was observed in S7, S11, and S17. All three also arrived at the same final comparison, $5/12 < 4/9$. However, these representational shifts and consistent outcomes were not accompanied by equally strong evidence of conceptual coordination.

For S7, conversion occurred from the symbolic to the visual register. After obtaining $15/36$ and $16/36$, S7 represented the fractions on a number line. The two representations were consistent in indicating that $5/12 < 4/9$. In the interview, S7 explained that the line was divided according to the denominator and that the fraction point was located according to the numerator (S7W48). S7 also explained that a larger value was located farther to the right (S7W49–S7W52) and related distance on the number line to the difference between the fraction values (S7W53–S7W58).

Nevertheless, the available evidence did not show an equally explicit explanation of how $5/12$ and $15/36$, and $4/9$ and $16/36$, preserve the same magnitude when expressed across different representations. The evidence for conceptual coordination in S7 was therefore categorized as partial within the context of the task.

S7 also demonstrated monitoring of the solution process. When asked what would be done if the calculation and number-line results differed, S7 stated that the calculation and drawing would be redone to determine the correct result (S7W18–S7W22). In the second interview, S7 also explained having experienced uncertainty when determining the least common multiple because of concern about making a calculation error (S7W43–S7W44).

In contrast, S11's conversion proceeded from the visual to the symbolic register. According to the interview, the number line was first used to obtain an answer, after which the common-denominator procedure was used to clarify the result. Both approaches ultimately produced the comparison $5/12 < 4/9$, indicating product-level consistency.

However, this consistency was not accompanied by a stable interpretation of the visual representation. When explaining why $4/9$ was located farther to the right than $5/12$, S11 stated, "because its value is smaller" (S11W40). When subsequently asked which fraction was smaller, S11 responded, "Umm, $4/9$ " (S11W44). These responses were inconsistent with the final comparison in the written work. S11 was also unable to provide a stable explanation of why intervals on the number line needed to be equal (S11W50–S11W52).

Accordingly, S11 demonstrated conversion and consistency in the final outcome, but conceptual coordination could not be established from the available evidence. This case shows that the use of two representations and agreement in the final result do not, by themselves, provide sufficient evidence that the quantitative relationship across representations has been understood consistently.

Monitoring in S11 was evident in corrections to the initial work and in the use of the symbolic procedure after the number line. When asked why the common-denominator procedure was still used after drawing the number line, S11 explained that it was used to make the answer "clearer" (S11W54). Thus, the symbolic procedure also served a confirmatory function for the initial result.

S17 also demonstrated conversion from symbolic to visual representation, but provided more explicit evidence of quantitative relationships. S17 linked $15/36$ with $5/12$ and $16/36$ with $4/9$ (S17W14–S17W18). In the second interview, S17 explained that the position of $5/12$ was obtained by dividing the unit interval from 0 to 1 into 12 equal parts and locating the fifth part (S17W52). When explaining the meaning of distance on the number line, S17 stated that "distance represents the quantity of the difference in value" (S17W56).

The combination of written work and verbal explanations provided strong evidence of conceptual coordination within the context of the analyzed task. This classification refers only to the strength of the evidence available for the task comparing $5/12$ and $4/9$, rather than to S17's representational ability in general.

S17 also used the number line as part of the monitoring process. When presented with the possibility that the calculation and visualization might yield different results, S17 stated that the calculation would be “done again” to determine the correct result (S17W24).

For S24, conversion and cross-representational coordination could not be assessed because only one register was used. The transformations $5/12 \rightarrow 15/36$ and $4/9 \rightarrow 16/36$ remained within the symbolic register and were therefore not categorized as conversion across registers.

Nevertheless, the interviews provided evidence concerning S24’s interpretation of the symbolic procedure. When asked why multiplying the numerator and denominator by the same number does not change the value of a fraction, S24 responded, “For that, Kak, I’m not really sure, hehe” (S24W20). In the second interview, S24 later stated that multiplying the numerator and denominator by the same number was “the same as multiplying by 1” (S24W72). However, when asked to elaborate on what remained unchanged, S24 was unable to explain further (S24W74).

S24’s written work also contained the justification that the fraction was smaller “because the numerator is smaller, so the result is larger” (see Fig. 4). This statement was not fully consistent with the correct comparison shown in the calculation. However, during the interview, S24 correctly explained that 15 is smaller than 16 (S24W10). This discrepancy was therefore treated as an inconsistency between the written justification and verbal explanation rather than as stand-alone evidence of a persistent misconception.

Across cases, the findings show that conversion, cross-representational consistency, and conceptual coordination are analytically distinct. S7, S11, and S17 all moved across representational registers and obtained consistent final outcomes, but the strength of evidence for coordination differed: partial for S7, not established from the available evidence for S11, and strong within the task context for S17. For S24, coordination across representations could not be assessed because only the symbolic register was used.

C. Final Warrant for Correctness

In addition to variation in representational functions and coordination, the interview analysis revealed a pattern in the representations or procedures students treated as the most decisive basis for accepting their answers as correct. Evidence concerning final warrant for correctness was interpreted cautiously, with both supporting and limiting evidence retained in the cross-case analysis.

For S7, this basis was not clearly articulated in the first interview. When presented with the possibility that the number-line result and the calculation might differ, S7 stated, “I don’t know which one would be more correct” (S7W16). In the second clarification interview, however, S7 stated, “I trust the calculation” (S7W60) because “the calculation is more certain

if it is correct" (S7W62). Near the end of the interview, when asked whether the answer would still be considered reliable without a drawing, S7 responded, "yes, as long as the calculation is correct" (S7W87).

Thus, the symbolic procedure as S7's final warrant became most explicit in the second clarification interview. The response from the first interview was retained as limiting evidence, indicating that this position had not been clearly articulated from the outset.

For S11, the number line was used as the initial strategy but was not treated as sufficient for confirming the answer. When asked whether the number line alone was sufficient, S11 responded, "not yet" (S11W58). When subsequently asked which approach was more trustworthy, S11 answered, "making the denominators the same" (S11W60). Thus, when the two representations used by S11 were compared, the symbolic procedure served as the more decisive basis for justification.

However, evidence that limited this interpretation was also present. When asked more generally how one could know whether an answer was correct, S11 responded, "I would ask my teacher" (S11W91). This response indicates an external source of confirmation in addition to mathematical representations. Because final warrant in this study was restricted to the mathematical representations or procedures used within the task, this response was not categorized as a representational final warrant but was retained as limiting evidence.

For S17, the symbolic basis for justification was stated explicitly. When asked which approach was more trustworthy, S17 selected the common-denominator procedure (S17W66). When asked to explain why, S17 stated that it was "more mathematically precise, as long as the steps are correct" (S17W70).

S17 is particularly informative because this case also showed the strongest evidence of conceptual coordination within the task context. Thus, stronger conceptual coordination did not automatically lead the number line to be treated as an equally decisive basis for justification.

S24 differed from the other three cases because no number line was used. The data therefore provided no basis for claiming that S24 preferred the symbolic representation over a visual one. The common-denominator procedure is more appropriately understood as the only procedural basis used in the solution. S24 stated that "the first thing is, we have to make the denominators the same and then compare" (S24W14) and explained that once the denominators were equal, the numerators were compared (S24W42).

For S24, the symbolic procedure was therefore the only observable procedural basis for obtaining and justifying the result. Because no alternative representation was used for comparison, the data were not used to establish a comparative final warrant or to infer a preference for symbolic over visual representation.

Across cases, a comparative final warrant could be established for S7, S11, and S17 because all three used both symbolic procedures and number lines and explicitly assigned greater justificatory weight to the symbolic procedure. For S24, final warrant could not be assessed comparatively because only the symbolic register was used; the symbolic procedure was therefore recorded as the only observable procedural basis.

Taken together, the cross-case findings indicate that, despite variation in representational functions and the strength of evidence for conceptual coordination, symbolic procedures remained central to students' acceptance of their answers as correct: they served as the explicitly more trusted basis in the three cases that used two representations and as the only procedural basis in the case that used only symbolic representation.

Discussion

The findings indicate that representational use in the task comparing $5/12$ and $4/9$ cannot be understood solely from the types of representations appearing in students' work. More informative distinctions concern the functions served by those representations, how students move between and connect meanings across representations, and which representation or procedure is ultimately treated as the basis for accepting an answer as correct. The cross-case analysis revealed three main patterns. First, the same representation served different functions across students. Second, conversion and consistency of outcomes did not, by themselves, indicate conceptual coordination. Third, among S7, S11, and S17, who used both symbolic procedures and number lines, symbolic procedures carried greater justificatory weight, whereas for S24, the symbolic procedure was the only procedural basis used.

A. Functions of Symbolic and Number-Line Representations Depend on How They Are Used

Variation in the functions of the number line across S7, S11, and S17 shows that the presence of a visual representation cannot be interpreted uniformly. For S11, the number line primarily functioned as a means of searching for or determining an initial answer. For S7, it was used after the symbolic solution to visualize fraction positions and verify the result. For S17, its function was broader, encompassing magnitude visualization, explanation of quantitative meaning, and evaluation of the plausibility of the result. S24 showed a different pattern by completing the task entirely within the symbolic register.

This pattern can be interpreted through Goldin's perspective [15], which positions representational systems as an important component of mathematical learning and problem solving. Within this perspective, external representations, formal notation, strategies, heuristics, and monitoring activities can be examined as observable mathematical behavior, whereas internal structures cannot be observed directly and must be inferred cautiously from

available behavior and contextual evidence. In the present study, this means that the function of a number line was not assigned simply because it appeared in a student's written work, but rather on the basis of the combined evidence from the sequence of written work and the student's explanation of its purpose.

Ainsworth's DeFT framework [17] provides a more specific basis for understanding these differences in the functions of multiple representations. Ainsworth identifies three broad functions of multiple external representations: complementing, constraining interpretation, and constructing deeper understanding. The benefits of multiple representations, however, depend on learners' ability to understand individual representations and, where necessary, the relationships among them. Learners may also treat representations separately and experience difficulty integrating information across them. From this perspective, the differing functions of the number line for S7, S11, and S17 illustrate that the potential of a representation is not necessarily identical to how it is actually used by a student.

The findings also add nuance to the potential benefits of number lines reported in previous research. Tian et al. [19] showed that number lines represent fractions as integrated magnitudes, whereas Patahuddin et al. [13] emphasized that features such as partitions, positions, intervals, and units function as mathematical affordances only when they are perceived and interpreted in mathematically appropriate relationships. The contrast between S11 and S17 illustrates this distinction: S11's use of the number line was not accompanied by a stable interpretation of position and magnitude, whereas S17 provided more explicit quantitative explanations of position and distance. The representational potential of the number line should therefore be distinguished from the quality of its actual use in students' solutions.

These findings suggest that analyses of multiple representations may overlook important information if students' work is classified only as "symbolic," "visual," or "using both." Across the four cases examined here, representational function provided a more informative distinction than the number or type of representations used.

B. Conversion, Cross-Representational Consistency, and Conceptual Coordination Are Distinct

The second finding highlights the need to distinguish three analytic dimensions: conversion, cross-representational consistency, and conceptual coordination. S7, S11, and S17 all used both symbolic and number-line registers and reached the same final relationship, $5/12 < 4/9$, yet the strength of evidence concerning relationships in meaning across representations differed among the three cases.

This distinction is grounded in Duval's framework [16]. Duval distinguishes two major forms of semiotic transformation: treatment and conversion. In the present study, the

transformations $5/12 \rightarrow 15/36$ and $4/9 \rightarrow 16/36$ were treated as treatments because they occurred within the same symbolic register. By contrast, movement from symbolic fractions to positions on a number line, or from a number line to a symbolic procedure, constituted conversion because it involved different representational registers. Duval further emphasizes coordination across registers as an important aspect of mathematical understanding.

The data show that successfully carrying out a conversion was not sufficient to establish conceptual coordination. For S11, movement from the visual to the symbolic register produced a consistent final outcome, but the interpretation of position and magnitude remained unstable, so conceptual coordination could not be established. S7 demonstrated relationships among partitioning, position, and distance but did not fully articulate how symbolic equivalence preserved magnitude across registers, resulting in partial evidence of coordination. S17 provided stronger evidence through explanations of unit partitioning, fraction position, and distance as a quantitative relationship. Together, these cases demonstrate that conversion, product-level consistency, and conceptual coordination should be treated as distinct analytic dimensions.

The comparison among S11, S7, and S17 therefore reveals at least three configurations: conversion with coordination that could not be established from the available evidence, conversion with partial evidence of coordination, and conversion with stronger evidence of coordination. This finding is consistent with Duval's emphasis that recognition of mathematical objects across different registers depends on coordination among registers rather than merely on the presence of more than one representation.

Accordingly, when multiple representations are analyzed, the more informative question is not simply "Did the student move to another representation?" but also "What quantitative relationship can the student explain as remaining invariant or corresponding across those representations?"

C. Final Warrant for Correctness and the Relationship Between Procedural and Conceptual Reasoning

The cross-case findings further show that representational function and justificatory function do not necessarily coincide. In this study, final warrant for correctness refers to the representation or procedure assigned the greatest justificatory weight when a student accepts an answer as correct within the given task. The construct is distinguished from the most frequently used strategy, the strategy perceived as easiest, representational preference, procedural accuracy, and level of conceptual understanding. For S7, S11, and S17, who used both symbolic procedures and number lines, evidence concerning final warrant was comparative: all three assigned greater justificatory weight to the symbolic procedure. For S7, the number line served visualization and verification functions, but final confidence was linked

to the correctness of the calculation. For S11, the number line was used to obtain an initial answer, whereas the common-denominator procedure was subsequently used to clarify the result and was considered more trustworthy. For S17, even though use of the number line was accompanied by stronger evidence of conceptual coordination, the common-denominator procedure was still regarded as more mathematically precise.

This pattern indicates that a representation may support solution search, visualization, explanation, or verification without becoming the most trusted basis for justification. In this sense, final warrant offers an additional analytic lens by directing attention not only to how a representation is used but also to the representation or procedure that is ultimately given greater weight in determining the validity of an answer. However, S11's statement that the student could "ask my teacher" when seeking confirmation indicates that sources of confidence need not be entirely representational. Accordingly, final warrant in this study is deliberately restricted to representational or procedural bases that arise within the task solution and is not intended to provide a complete account of students' sources of epistemic authority.

These findings should not be extended to imply that symbolic representations have a generally higher mathematical or epistemic status than visual representations. The observed pattern indicates only how S7, S11, and S17 treated the two forms of representation within this particular task. For S24, such a comparison could not be made because only the symbolic register was used. The common-denominator procedure in this case is therefore more appropriately interpreted as the only observable procedural basis rather than as evidence of a preference for symbolic over visual representation.

The comparison between S17 and S24 also illustrates that procedural success and the quality of conceptual explanation do not necessarily align in the same way. S24 obtained the correct comparison through the common-denominator procedure but had difficulty explaining why multiplying both the numerator and denominator by the same number preserves the value of a fraction, while the written justification was also not fully consistent with the correct calculation. By contrast, S17 provided more explicit explanations of position, intervals, and distance on the number line while still assigning greater final justificatory weight to the symbolic procedure. This pattern can be interpreted cautiously through Skemp's distinction [23] between instrumental understanding and relational understanding, without classifying students as having fixed types of understanding. It is also consistent with Lenz et al. [18], who showed that procedural and conceptual knowledge of fractions can be distinguished empirically while remaining interrelated. Thus, successful execution of a common-denominator procedure does not, by itself, provide equivalent information about the quality of the conceptual understanding accompanying that procedure.

The use of the common-denominator procedure as a prominent strategy also bears some resemblance to the findings of Youkap and Nguala [20], who reported systematic reliance on a single procedure among preservice primary teachers across several fraction-comparison tasks and related this pattern to limited procedural flexibility. However, differences in participants and research design constrain this comparison. The present findings therefore do not support a general claim of procedural rigidity among the students studied; rather, they indicate differing degrees of reliance on the common-denominator procedure within the particular task analyzed.

D. Implications for Representational Tasks and Assessment in Fraction Learning

The findings have implications for how representational use is assessed in fraction learning. Asking students to use more than one representation may provide teachers with richer information, but the presence of two representations alone is insufficient to demonstrate that students understand the mathematical relationship between them. Research on fraction comparison has shown substantial variation in students' choice and successful implementation of strategies, indicating that producing a visual or symbolic strategy does not necessarily imply successful reasoning about fraction magnitude [24]. Similarly, research on representational translation emphasizes that successful coordination requires students to unpack a source representation, construct a target representation, and establish equivalence between them [25]. Representational assessment can therefore move beyond prompts such as "use another method" or "draw a number line" by asking students to explain why two representations show the same value, why intervals on a number line must be equal, what remains invariant when a fraction is expressed in an equivalent form, and how the result from one representation can be used to evaluate the result from another. Such questions can help distinguish the ability to produce representations from the ability to coordinate meaning across representations [25], [26].

This distinction is particularly important for number-line tasks. Although number lines can support fraction magnitude understanding, students must coordinate fraction concepts with the spatial and numerical structure of the number line [26], [27]. Studies have consequently found substantial variation in students' understanding of fraction placement and interpretation on number lines [26]. Thus, successful production of a number-line representation should not automatically be interpreted as evidence of fully coordinated fraction understanding.

The finding concerning final warrant also suggests potential value in asking questions such as, "Which method do you trust most, and why?" Such questions may reveal justificatory dimensions that are not always visible in written products. Research on fraction comparison indicates that students may employ different strategies—including visual models, common

denominators, benchmarks, or component-based reasoning—with different levels of success [24], [28]. For S7 and S17, for example, the number line served meaningful functions in their solutions, yet the symbolic procedure remained the more trusted basis. This type of information may help teachers examine not only which strategies students use but also the basis on which they evaluate the validity of their mathematical answers.

These implications do not suggest that symbolic procedures should be reduced or replaced by number lines. Rather, the findings point to the importance of connecting symbolic procedures with the quantitative meanings they represent. Recent work on fraction learning supports combining opportunities for conceptual exploration with structured procedural practice rather than treating the two forms of knowledge as competing instructional goals [29]. Similarly, number-line instruction can support students in connecting fraction symbols, unit fractions, magnitude, and arithmetic relationships when the representation is explicitly linked to mathematical meaning [27], [30]. Students should therefore not only be able to carry out the common-denominator procedure but also explain why it works, while number lines should function as more than supplementary drawings by supporting explanations of position, intervals, equivalence, and magnitude.

Fraction-comparison tasks can therefore be designed so that students not only obtain an answer but also compare representations, explain relationships among them, examine the consistency of results, and articulate why a particular result can be accepted as correct. Research on multiple-representation instruction similarly emphasizes that merely presenting several representations is insufficient; explicit attention must be given to how mathematical elements correspond across representations [31], [32]. Assessment frameworks for representational translation also support examining students' ability to move between, coordinate, and establish equivalence among representational forms rather than evaluating each representation independently [25], [33]. These implications remain analytic and pedagogical; the present study did not test the effectiveness of any particular instructional method or intervention.

E. Limits of Interpretation and Contributions of the Study

The findings should be interpreted within the boundaries of the study design. The in-depth analysis focused on four cases from a single classroom and was based on one task comparing two fractions. The patterns identified are therefore not intended to represent all students, all types of fraction-comparison tasks, or mathematical representational use more generally. This qualification is especially important because strategy selection in fraction comparison can vary according to task characteristics, students' knowledge, and the numerical properties of the fractions being compared [24], [28].

In addition, the number line was explicitly mentioned as an optional representation in the task instructions. Its use therefore cannot be treated as having emerged entirely spontaneously without prompting. Differences in how the number line functioned can still be analyzed, but the study cannot determine whether the same students would have selected a number line had it not been mentioned in the instructions. This distinction matters because studies of multiple representations indicate that learners' representational choices and their capacity to coordinate representations can be influenced by task design and the representational options made available [32], [34].

The conditions under which the written task was completed provide another limit on interpretation. Although students were instructed to work individually, some engaged in brief discussions about the problem, strategies, or answers during task completion. Strategies appearing in the written work therefore cannot be assumed to have developed entirely independently of peer interaction. This limitation primarily affects interpretations concerning the origins of particular strategies, although the functions, reasons for use, and justificatory bases associated with the representations used by the four focal cases were subsequently clarified through individual interviews.

The interviews were also retrospective in relation to students' written work. Interview data were therefore treated as evidence of how students explained, evaluated, and justified the strategies and representations they had used, rather than as direct access to the mental processes occurring during initial task completion. This limitation is consistent with Goldin's caution [15] that internal structures and processes cannot be observed directly but can only be inferred from observable behavior, interviews, notation use, and contextual information. Accordingly, claims in the present study are deliberately restricted to reasoning and representational use that can be traced through students' written work and verbal explanations.

The study also did not directly observe classroom instruction or compare instructional conditions. Although the students had previously encountered several fraction-comparison strategies, the available data do not support the conclusion that the observed priority given to symbolic procedures was caused by procedurally oriented instruction. Research indicates that conceptual and procedural fraction knowledge can vary substantially even within the same learner and across fraction topics, suggesting that observed performance should not automatically be attributed to a single instructional orientation [29], [35]. The patterns identified should therefore be understood as representational and justificatory patterns evident within the analyzed task rather than as causal outcomes of particular instructional practices.

Within these boundaries, the contribution of the study lies not in proposing a general developmental model of fraction understanding, but in three analytic distinctions emerging from the cross-case comparison. First, representational form should be distinguished from representational function: the same number-line representation may serve initial solution search, visualization, explanation, or verification depending on how it is used. Research on number-line learning similarly indicates that visual representations can function as tools for sense-making only when learners coordinate their structural properties with the relevant mathematical relationships [27], [36].

Second, conversion, consistency of outcomes, and conceptual coordination should be treated as distinct dimensions: students may move across registers and obtain the same result without sufficient evidence that the quantitative relationship across representations has been coordinated. Research on translation among representations explicitly distinguishes movement between representations from establishing equivalence between source and target representations [25], while studies of classroom representation use similarly emphasize the need to make connections across corresponding conceptual elements rather than merely translating surface forms [31].

Third, representational function can be distinguished from justificatory function. In the three cases that used both symbolic and number-line representations, the symbolic procedure served as the final warrant for correctness even though the number line fulfilled different representational functions. For S24, who used only the symbolic register, the symbolic procedure constituted the only procedural basis and therefore did not support a hierarchical comparison between symbolic and visual representations. This distinction is compatible with evidence showing that learners can possess several available representations or strategies without integrating them into a flexible network of rational-number knowledge [30].

Accordingly, final warrant for correctness is not proposed here as a general theory of the epistemic status of mathematical representations. Rather, it is used as a task-specific analytic lens for identifying the representation or procedure that students treat as the most decisive basis for accepting an answer as correct. Within the bounds of these four cases and this single task, final warrant complements analyses of representational function and coordination by directing attention not only to what students use and how they connect representations, but also to what they ultimately treat as the basis for saying that an answer is correct.

Future research could examine whether these patterns recur across fraction-comparison tasks with different numerical characteristics, when number lines are not explicitly mentioned in the instructions, or when students encounter other visual representations. This is important because comparison strategies and cognitive processes can change according to fraction structure and task characteristics [24], [28]. Using multiple tasks would also make it possible

to examine whether final warrant remains relatively stable or varies with problem characteristics.

Longitudinal studies and classroom observations could further investigate how such justificatory bases develop and how instructional experiences relate to the ways students use and assign weight to different representations. Recent studies suggest that carefully aligned number-line experiences can promote transfer in fraction understanding [37], while classroom-based research indicates that sustained work with number lines and unit fractions can provide productive connections among fraction ideas [36]. Such work could determine whether the representational and justificatory distinctions identified here remain stable over time or change as students develop more flexible fraction knowledge.

Conclusion

This study shows that the use of symbolic and number-line representations in the task comparing $5/12$ and $4/9$ should be understood in terms of representational function rather than solely in terms of the representational forms that appear. Across the four cases analyzed, the number line served different functions, ranging from initial solution search to visualization, explanation, and verification, whereas symbolic procedures were used to obtain, clarify, or justify results. Thus, the presence of the same representation does not necessarily indicate the same reasoning function.

The findings also show that conversion, cross-representational consistency, and conceptual coordination are analytically distinct. S7, S11, and S17 all moved between symbolic and number-line representations and arrived at consistent final outcomes, but the strength of evidence for coordination differed: it was partial for S7, could not be established from the available evidence for S11, and was stronger within the task context for S17. For S24, coordination across representations could not be assessed because only the symbolic register was used. Therefore, the ability to produce or move between multiple representations cannot automatically be treated as evidence that the quantitative relationships across those representations have been conceptually coordinated.

The cross-case analysis further indicates that representational function does not necessarily coincide with justificatory function. For S7, S11, and S17, the common-denominator procedure carried greater final justificatory weight than the number line, whereas for S24, the symbolic procedure was the only procedural basis used. This finding supports the use of final warrant for correctness as a task-specific analytic lens: in addition to examining which representations students use and how they connect them, analysis can also consider which representation or procedure students ultimately treat as the basis for accepting an answer as correct. This construct is not intended as a general theory of the superiority or epistemic status of any particular type of representation.

The contribution of this study therefore lies in three analytic distinctions: representational form versus representational function; conversion and consistency of outcomes versus conceptual coordination; and representational function versus justificatory function. These findings should be interpreted within the boundaries of four cases from a single classroom and one fraction-comparison task, with the number line explicitly mentioned as an optional representation in the task instructions and the interviews conducted retrospectively. Accordingly, the study describes observable patterns of reasoning and justification within the task context analyzed rather than students' general cognitive characteristics.

Conflict of Interest

The author declares that there are no conflicts of interest regarding the conduct of this research or the writing of this article.

References

- [1] S. Getenet and R. Callingham, "Teaching fractions for understanding: Addressing interrelated concepts," in *40 Years On: We Are Still Learning! Proc. 40th Annu. Conf. Math. Educ. Res. Group Australasia*, A. Downton, S. Livy, and J. Hall, Eds. Melbourne, Australia: MERGA, 2017, pp. 277–284.
- [2] R. Novita, T. Herman, D. Dasari, and M. Putra, "Analyzing second-year university students' rational number understanding: A case on interpreting and representing fraction," *Eur. J. Educ. Res.*, vol. 11, no. 3, pp. 1747–1762, 2022, doi: 10.12973/eu-jer.11.3.1747.
- [3] R. S. Siegler, C. A. Thompson, and M. Schneider, "An integrated theory of whole number and fractions development," *Cogn. Psychol.*, vol. 62, no. 4, pp. 273–296, 2011, doi: 10.1016/j.cogpsych.2011.03.001.
- [4] R. S. Siegler, L. K. Fazio, D. H. Bailey, and X. Zhou, "Fractions: The new frontier for theories of numerical development," *Trends Cogn. Sci.*, vol. 17, no. 1, pp. 13–19, 2013, doi: 10.1016/j.tics.2012.11.004.
- [5] S. A. Syed Ismail, S. M. Maat, and F. Khalid, "35 years of fraction learning: Integrating systematic review and bibliometric analysis on a global scale," *Eurasia J. Math. Sci. Technol. Educ.*, vol. 20, no. 12, Art. no. em2543, 2024, doi: 10.29333/ejmste/15657.
- [6] F. Aliustaoğlu, A. Tuna, and A. Ç. Biber, "The misconceptions of sixth grade secondary school students on fractions," *Int. Electron. J. Elem. Educ.*, vol. 10, no. 5, pp. 591–599, 2018, doi: 10.26822/iejee.2018541308.
- [7] Y. Deringöl, "Misconceptions of primary school students about the subject of fractions: Views of primary teachers and primary pre-service teachers," *Int. J. Eval. Res. Educ.*, vol. 8, no. 1, pp. 29–38, 2019, doi: 10.11591/ijere.v8i1.16290.
- [8] H. Herliana, M. Maison, and S. Syaiful, "Development and implementation of a five-tier diagnostic test to identify student misconceptions on fractions: A significant step towards improving mathematics education," *J. Ilm. Ilmu Terap. Univ. Jambi*, vol. 8, no. 2, pp. 563–576, 2024, doi: 10.22437/jiituj.v8i2.34159.
- [9] S. Alonso-Díaz, S. T. Piantadosi, B. Y. Hayden, and J. F. Cantlon, "Intrinsic whole number bias in humans," *J. Exp. Psychol. Hum. Percept. Perform.*, vol. 44, no. 9, pp. 1472–1481, 2018, doi: 10.1037/xhp0000544.
- [10] A. Altan and Z. T. Sener, "Developing the diagnostic test of misconceptions of fractions," in *Proc. EJR Congress 2023 Int. Eurasian Educational Research Congress*, Ankara, Türkiye: Ani Publishing, 2023, pp. 255–272.
- [11] R. A. Ayieko, G. Moreano, and L. Harter, "A cross-national comparison of fourth and eighth grade students' understanding of fraction magnitude," *Int. Electron. J. Math. Educ.*, vol. 17, no. 4, Art. no. em0703, 2022, doi: 10.29333/iejme/12287.

- [12] J. Rodrigues, S. Locke, E. L. Singell, and L. G. Mirielli, "Teaching fraction magnitude using the number line," *Interv. Sch. Clin.*, vol. 59, no. 3, pp. 165–172, 2024, doi: 10.1177/10534512231156885.
- [13] S. M. Patahuddin, H. B. Usman, and A. Ramful, "Affordances from number lines in fractions instruction: Students' interpretation of teacher's intentions," *Int. J. Sci. Math. Educ.*, vol. 16, no. 5, pp. 909–928, 2018, doi: 10.1007/s10763-017-9800-z.
- [14] S. Sirajuddin, I. Akib, and N. Nasrun, "Visualizing fractions: Enhancing problem-solving performance through diagrammatic reasoning in elementary mathematics," *J. Penelit. Pengkaj. Ilmu Pendid. e-Saintika*, vol. 9, no. 2, pp. 512–524, 2025, doi: 10.36312/e-saintika.v9i2.3139.
- [15] G. A. Goldin, "Representational systems, learning, and problem solving in mathematics," *J. Math. Behav.*, vol. 17, no. 2, pp. 137–165, 1998, doi: 10.1016/S0364-0213(99)80056-1.
- [16] R. Duval, "A cognitive analysis of problems of comprehension in a learning of mathematics," *Educ. Stud. Math.*, vol. 61, no. 1–2, pp. 103–131, 2006, doi: 10.1007/s10649-006-0400-z.
- [17] S. Ainsworth, "DeFT: A conceptual framework for considering learning with multiple representations," *Learn. Instr.*, vol. 16, no. 3, pp. 183–198, 2006, doi: 10.1016/j.learninstruc.2006.03.001.
- [18] K. Lenz, A. Dreher, L. Holzäpfel, and G. Wittmann, "Are conceptual knowledge and procedural knowledge empirically separable? The case of fractions," *Br. J. Educ. Psychol.*, vol. 90, no. 3, pp. 809–829, 2020, doi: 10.1111/bjep.12333.
- [19] J. Tian, V. Bartek, M. Z. Rahman, and E. A. Gunderson, "Learning improper fractions with the number line and the area model," *J. Cogn. Dev.*, vol. 22, no. 2, pp. 305–327, 2021, doi: 10.1080/15248372.2021.1890603.
- [20] P. T. Youkap and J.-B. Nguala, "Understanding and strategies for comparing fractions among pre-service teachers: Between procedural rigidity and conceptual flexibility," *Int. Electron. J. Math. Educ.*, vol. 20, no. 4, Art. no. em0849, 2025, doi: 10.29333/iejme/16834.
- [21] G. Pólya, *How to Solve It: A New Aspect of Mathematical Method*, 2nd ed. Princeton, NJ, USA: Princeton University Press, 1957.
- [22] V. Braun and V. Clarke, "Using thematic analysis in psychology," *Qual. Res. Psychol.*, vol. 3, no. 2, pp. 77–101, 2006, doi: 10.1191/1478088706qp063oa.
- [23] R. R. Skemp, *The Psychology of Learning Mathematics: Expanded American Edition*. Hillsdale, NJ, USA: Lawrence Erlbaum Associates, 1987.
- [24] J. Liu and E. Jacobson, "Examining US elementary students' strategies for comparing fractions after the adoption of the Common Core State Standards for Mathematics," *J. Math. Behav.*, vol. 67, Art. no. 100985, 2022, doi: 10.1016/j.jmathb.2022.100985.
- [25] P. Andriani, K. R. A. Kurniawati, and D. Afriyani, "A framework for assessing translation among multiple representations," *JTAM (J. Teori Apl. Mat.)*, vol. 6, no. 2, pp. 321–330, 2022, doi: 10.31764/jtam.v6i2.7193.
- [26] J. Kim, "Profiles of young students' understanding of fractions on number lines," *Eurasia J. Math. Sci. Technol. Educ.*, vol. 20, no. 5, Art. no. em2444, 2024, doi: 10.29333/ejmste/14469.
- [27] T. Lesner, M. Sutherland, C. Lussier, and B. Clarke, "Using the number line to build understanding of fraction arithmetic," *Interv. Sch. Clin.*, vol. 59, no. 3, pp. 191–198, 2024, doi: 10.1177/10534512231156878.
- [28] F. Reinhold, T. Leuders, and K. Loibl, "Disentangling magnitude processing, natural number biases, and benchmarking in fraction comparison tasks: A person-centered Bayesian classification approach," *Contemp. Educ. Psychol.*, vol. 75, Art. no. 102224, 2023, doi: 10.1016/j.cedpsych.2023.102224.
- [29] M. Mavrikis, N. Rummel, M. Wiedmann, K. Loibl, and W. Holmes, "Combining exploratory learning with structured practice educational technologies to foster both conceptual and procedural fractions knowledge," *Educ. Technol. Res. Dev.*, vol. 70, pp. 691–712, 2022, doi: 10.1007/s11423-022-10104-0.
- [30] J. McMullen, A. Koskinen, T. Karki, A. Lindstedt, S. Maatta, H. Halme, E. Lehtinen, M. M. Hannula-Sormunen, and K. Kiili, "A game-based approach to promoting adaptive rational number knowledge," *Math. Think. Learn.*, vol. 26, no. 4, pp. 411–427, 2024, doi: 10.1080/10986065.2023.2177818.
- [31] M. Post and S. Prediger, "Teaching practices for unfolding information and connecting multiple representations: The case of conditional probability information," *Math. Educ. Res. J.*, vol. 36, pp. 97–129, 2024, doi: 10.1007/s13394-022-00431-z.

- [32] G. A. Nigusse and K. Michael, "GeoGebra supported multiple representations to enhance representational skills in calculus," *Asian J. Assess. Teach. Learn.*, vol. 12, no. 2, pp. 110–121, 2022, doi: 10.37134/ajatel.vol12.2.10.2022.
- [33] Y. A. Putri and F. Q. Aini, "Development of a test instrument using multiple representations to assess students understanding: A Rasch model analysis," *J. Pijar MIPA*, vol. 18, no. 2, pp. 183–193, 2023, doi: 10.29303/jpm.v18i2.4721.
- [34] R. N. Yuliandari, C. Sa'dijah, Susiswo, and Purnomo, "Teachers' multiple representations on teaching fractions at elementary school: A commognitive framework," *AL-ISHLAH: J. Pendidik.*, vol. 16, no. 2, pp. 1005–1018, 2024, doi: 10.35445/alishlah.v16i2.5048.
- [35] K. Lenz, F. Reinhold, and G. Wittmann, "Topic specificity of students' conceptual and procedural fraction knowledge and its impact on errors," *Res. Math. Educ.*, vol. 26, no. 1, pp. 45–69, 2024, doi: 10.1080/14794802.2022.2135132.
- [36] C. D. Bruce, T. Flynn, S. Yearley, and Z. Hawes, "Leveraging number lines and unit fractions to build student understanding: Insights from a mixed methods study," *Can. J. Sci. Math. Technol. Educ.*, vol. 23, pp. 322–339, 2023, doi: 10.1007/s42330-023-00278-x.
- [37] S. Yu, P. Sidney, D. Kim, C. A. Thompson, and J. E. Opfer, "From integers to fractions: The role of analogy in transfer and long-term learning," *J. Exp. Child Psychol.*, vol. 243, Art. no. 105918, 2024, doi: 10.1016/j.jecp.2024.105918.

Authors



Miftahul Jannah, S.Pd. is a master's student in Mathematics Education at Universitas Negeri Malang, Indonesia. Her academic interests include mathematical representation, mathematical cognition, problem solving, and mathematics education. Her current work focuses on how students use and coordinate multiple representations when reasoning about mathematical concepts, particularly fractions. (email: miftahul.jannah.2503118@students.um.ac.id).



Dr. Rustanto Rahardi, M. Si.   is a lecturer in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Malang, Indonesia. His academic activities involve teaching and research in mathematics and mathematics education. He is also engaged in scholarly publication and academic supervision within the department. (email: rustanto.rahardi.fmipa@um.ac.id).



Dr. Dahliatul Hasanah, S.Si., M. Math.Sc.    is a lecturer in the Department of Mathematics, Universitas Negeri Malang, Indonesia. Her areas of expertise include analysis, applied mathematics, and mathematics education. She is actively involved in research, academic publication, and international scholarly activities, and has served as Chair of The International Conference on Mathematics and Its Applications. (email: dahliatul.hasanah.fmipa@um.ac.id).