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The Combined Approach of Teaching at the Right Level and Problem-Based Learning to Foster Mathematical Critical Thinking Skills

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ABSTRACT

Mathematical critical thinking is an essential skill that helps students analyze information, evaluate arguments, and solve complex problems systematically. However, many elementary students still struggle to attain higher-order thinking skills, mainly due to learning gaps in the classroom. This study aims to examine the effect of the integrated Teaching at the Right Level (TaRL) and Problem-Based Learning (PBL) approach on improving students' mathematical critical thinking skills. This research used a quasi-experimental method involving 34 fourth-grade students divided into experimental and control groups. The experimental group received TaRL–PBL instruction, while the control group used PBL only. Critical thinking skills were measured using pretest and posttest essay questions based on five indicators: interpretation, analysis, inference, evaluation, and explanation. The results showed a significant improvement in the experimental group ($t = 2.462$, $p < 0.05$) with a large effect size (Cohen's $d = 0.846$), indicating that the TaRL–PBL approach had a strong impact on enhancing critical thinking. These findings suggest that integrating level-based instruction and problem-solving is effective in supporting differentiated learning and strengthening critical thinking in mathematics. The study also indicates that this integrated approach has strong potential to be further developed in mathematics education, particularly to enhance other skills such as creative thinking and mathematical connections.

Keywords
Critical Thinking
Diagnostic Assessment
Elementary Students
Problem-Based Learning
Teaching at the Right Level

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Introduction

Mathematical critical thinking is a higher-order thinking skill that students must possess. This skill helps students analyze information, evaluate arguments, and solve mathematical problems logically and systematically [1]–[3]. It emphasizes not only computational ability but also reasoning, interpretation, conceptual linking, and evidence-based decision-making [4]–[6]. Students with mathematical critical thinking skills can interpret, analyze, infer, evaluate, and explain when solving problems [7]–[9]. This skill is essential in the 21st century, where learners are expected to think analytically, reflectively, and solve complex problems [10].

However, Indonesian students' performance in this area remains low. According to the PISA report, only about 30% of Indonesian students reach the basic level of mathematical literacy, far below the OECD average [11]. TIMSS findings also indicate that most Indonesian students struggle with math problems requiring critical thinking [12]. Students lacking this skill tend to face difficulties in understanding concepts, solving problems, and making decisions [13].

Students' inability to think critically is also influenced by the diversity of competency levels in each classroom. There is a significant gap between students who have mastered the material and those who struggle with basic concepts. This leads some students to fall behind and lose motivation, while others feel unchallenged [14].

Therefore, innovative instructional approaches are needed to address varying student abilities and stimulate critical thinking skills. One such approach is Teaching at the Right Level (TaRL). TaRL emphasizes the importance of aligning instruction with students' actual competence levels. It helps teachers identify student abilities and deliver targeted instruction to each group [15], [16].

A potential model to enhance critical thinking is Problem-Based Learning (PBL). This approach emphasizes solving authentic problems to develop student knowledge and skills. The learning process in PBL generally begins with the presentation of a problem, followed by organizing students to engage in the learning process. Teachers then guide both individual and group investigations before students proceed to develop and present their work [17], [18].

Through these stages, students are trained to think critically and apply mathematical concepts in real-world contexts [19], [20].

Teaching at the Right Level (TaRL) and Problem-Based Learning (PBL) have been widely applied in various countries, reflecting their global relevance in improving student learning outcomes. In India, TaRL has been effectively used to reduce learning gaps in mathematics by grouping students according to their current ability levels, which allows teachers to provide more targeted instruction and foster deeper understanding of basic concepts [21,22] . Similar practices have been adapted in African countries such as Kenya and Nigeria, where TaRL has contributed to improving foundational numeracy and enabling students to progress to more complex mathematical reasoning [23]. Meanwhile, PBL has been extensively implemented in diverse contexts, including mathematics classrooms in Europe and Southeast Asia, where authentic problem-solving activities have been shown to enhance students' analytical abilities and higher-order thinking skills [24]-[26]. Collectively, these findings demonstrate that TaRL provides the scaffolding necessary for building a solid foundation in mathematics, while PBL encourages students to engage in reasoning, analysis, and evaluation. Both approaches therefore play a complementary role in nurturing students' mathematical critical thinking, which is essential for solving real-world problems and developing adaptive expertise.

The integration of TaRL and PBL is believed to create adaptive, challenging, and meaningful mathematics learning, while effectively supporting the development of students' critical thinking skills. Although TaRL and PBL have been implemented separately with positive outcomes, their combined application remains underexplored. This study aims to examine the effect of the integrated TaRL and PBL approach on improving elementary students' critical thinking skills.

Material and Methods

This study used a quantitative research method with a quasi-experimental design. The design applied was a non-equivalent control group. It involved two groups: an experimental group and a control group. Before treatment, both groups took a pretest to measure their initial condition. The experimental group received special treatment using the TaRL and PBL approach, while the control group received instructions using PBL only. Afterward, both groups took a posttest to measure their average mathematical critical thinking skills.

The design of the experimental group received special treatment using the TaRL and PBL is presented in Fig. 1.

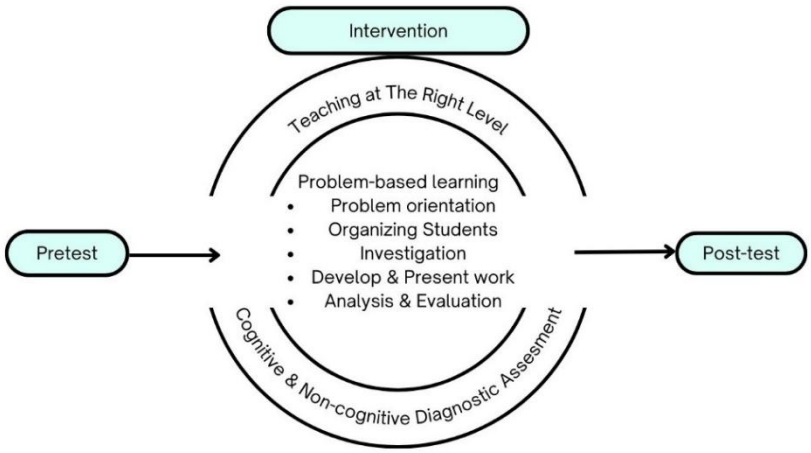


Fig. 1. Learning design using TaRL and PBL

The learning design unfolds in five phases that integrate TaRL grouping with PBL cycles:

1. Diagnostic Assessment and Grouping (TaRL stage)
   - Students are assessed through short diagnostic tests to identify their current level of mathematical understanding.
   - Based on results, students are grouped into *foundational*, *intermediate*, and *advanced* categories.
2. Problem Orientation (PBL stage)
   - Each group is introduced to authentic mathematical problems that are tailored to their level.
3. Organization for Learning (TaRL + PBL stage)
   - Within each group, students organize roles and learning strategies collaboratively.
   - Teachers act as facilitators, ensuring that group tasks remain aligned with the students' readiness levels.
4. Guided Investigation and Collaboration (PBL stage with TaRL scaffolding)
   - Students engage in individual exploration followed by group discussions.
   - Teachers provide scaffolding aligned with each group's ability, ensuring progression from concrete strategies (foundational) to abstract reasoning (advanced).
5. Presentation, Analysis, and Evaluation (PBL stage)
   - Groups present their solutions to the class.
   - The teacher guides reflection sessions where students compare different approaches, analyze strengths and weaknesses, and connect solutions across ability levels.
   - Evaluation includes both peer feedback and teacher assessment of critical thinking indicators.

The field research was conducted from March to May 2025 at a private elementary school in Sleman, Yogyakarta. The study population consisted of 34 fourth-grade students divided into Class A and Class B. These two classes were randomly assigned as experimental and control groups. Based on the draw, Class A with 19 students was selected as the experimental group, and Class B with 15 students as the control group.

The data collection instruments were pretest and posttest questions measuring critical thinking skills. The questions were developed based on the theory of mathematical critical thinking. A total of 5 pretest and 5 posttest questions were prepared in essay format. The aspects of critical thinking skills are presented in Table 1.

Table 1. Aspects of critical thinking skills

Aspect	Indicator
Interpretation	Students can interpret important information in area measurement using non-standard units.
Analysis	Students can analyze volume using standard and non-standard units.
Inference	Students can draw conclusions from the results of volume measurement using standard and non-standard units.
Evaluation	Students can evaluate the accuracy of area measurement results using standard and non-standard units.
Explanation	Students can explain measurement strategies using non-standard units.

The developed instrument was tested for validity and reliability. The instrument was tested on 30 students from different schools. The trial results were used to assess the instrument's validity and reliability. The validity test was conducted using SPSS 25 at a 5% significance level. The results are shown in Table 2.

Table 2. Validity Test Results – Pretest and Posttest

No	r-calculated		r table	Description
	Pre test	Post test		
1	0.742	0.698	0.388	valid
2	0.742	0.707	0.388	valid
3	0.746	0.631	0.388	valid
4	0.501	0.640	0.388	valid
5	0.590	0.730	0.388	valid

Based on the SPSS output in Table 2, the r-table value was 0.388. Since all r-calculated values were greater than r-table, it can be concluded that the pretest and posttest items were valid and could be used to measure critical thinking skills. The reliability test results are presented in Table 3.

Table 3. Reliability Test Results

Instrument	Cronbach's Alpha	N of items
Pre test	0.726	5
Post test	0.709	5

Instrument reliability was evaluated using Cronbach's Alpha. Based on Table 3, the Cronbach's Alpha score for the pretest instrument was 0.726, and for the posttest instrument, it was 0.709. Both values exceeded the minimum acceptable threshold of 0.70, indicating that the instrument had strong internal consistency and was reliable for measuring mathematical critical thinking skills.

The data analysis techniques used in this study were descriptive and inferential analysis. Descriptive analysis was used to examine the average score of students' critical thinking skills. Each indicator score was 20 points, with a maximum total score of 100 and a minimum of 0. Table 4 shows the score categories.

Table 4. Score range

Interval	Interval of critical thinking skills	Description
$MI - 3\ SDI \leq X \leq MI - 1.5\ SDI$	$0 \leq X \leq 25$	Very low
$MI - 1.5\ SDI < X \leq MI - 0.5\ SDI$	$25 < X \leq 41.67$	Low
$MI - 0.5\ SDI < X \leq MI + 0.5\ SDI$	$41.67 < X \leq 58.33$	Moderate
$MI + 0.5\ SDI < X \leq MI + 1.5\ SDI$	$58.33 < X \leq 75$	High
$MI + 1.5\ SDI < X \leq MI + 3\ SDI$	$75 < X \leq 100$	Very high

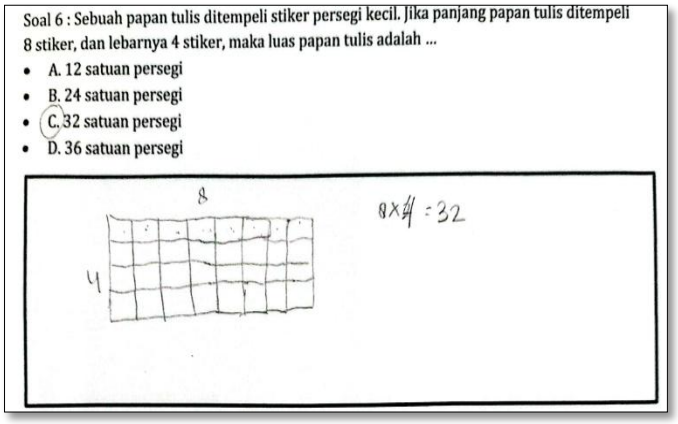
The inferential analysis was used to test the research hypothesis regarding the effect of the TaRL and PBL integration on students' critical thinking skills. The hypothesis was tested using an independent sample t-test, assisted by SPSS 25.

Results and Discussions

The learning activities were conducted in five meetings for both the experimental and control classes. These activities were held outside the pretest and posttest sessions.

A. Diagnostic Assessment

In the experimental class, the implementation of the TaRL approach began with cognitive and non-cognitive diagnostic assessments to identify students' initial abilities and learning preferences. The diagnostic assessment was developed based on the guidelines from the cognitive diagnostic assessment handbook [27]. The cognitive assessment consisted of 10 questions covering prerequisite material, specifically measurement and geometry. The 10 questions included two from the Grade 2 second semester learning outcomes, six from Grade 3, and two related to the upcoming material. The purpose of this assessment was to align the learning process with the students' actual condition. Thus, students were expected to master the prerequisite material before learning new content. One example of a student's work on the cognitive diagnostic assessment is presented in Fig. 2.



The translation:
Question: A blackboard is covered with small square stickers. If the length of the blackboard is covered with 8 stickers and the width with 4 stickers, then the area of the blackboard is …
A. 12 square units
B. 24 square units
C. 32 square units
D. 36 square units

Fig. 2. Student's response of cognitive diagnostic assessment

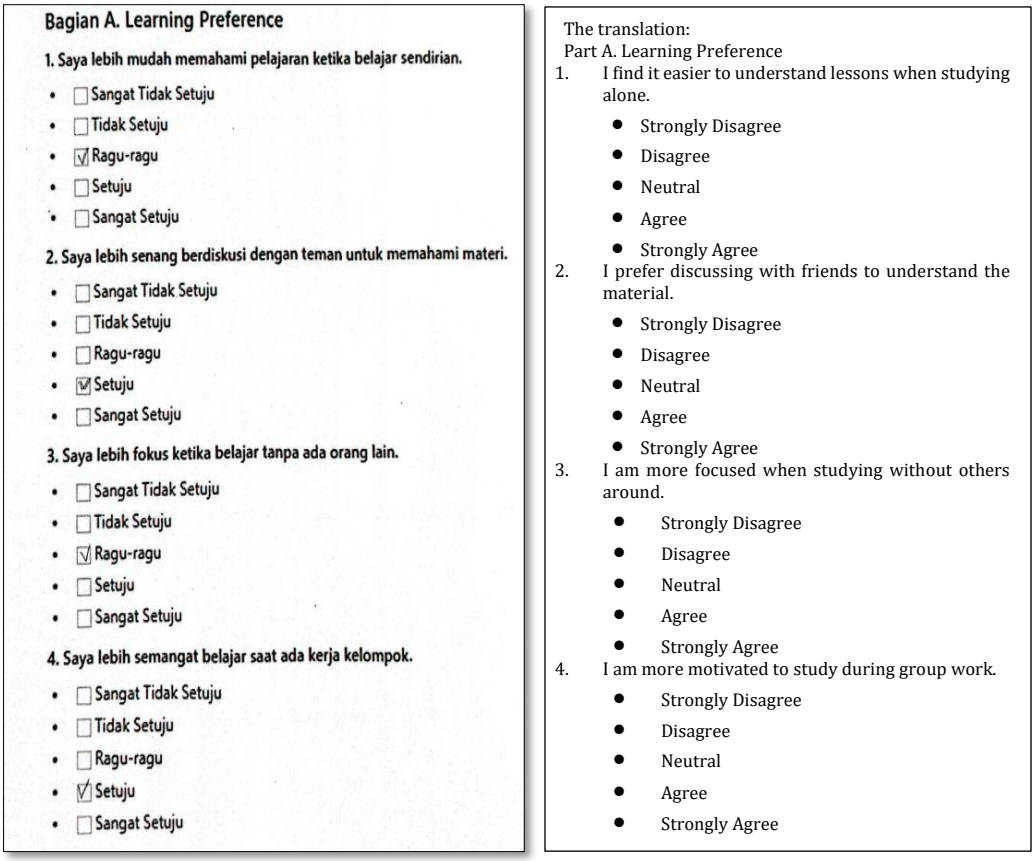
Fig. 2 shows the student's response in the cognitive diagnostic assessment for the learning objective *measuring area using non-standard units*. The students' work demonstrates that they already possess the ability to measure the surface area of objects using non-standard units. This is evident from the way the student illustrated the board and divided it into the units given in the task. In addition, the student also understood that the area of the board could be measured using the formula for the area of a rectangle. A summary of the cognitive diagnostic assessment results is presented in Table 5.

Table 5. Result of cognitive diagnostic assesment

Learning Objective	Question Number	Number of Students Answering	
		Correct	Incorrect
Identifying various plane shapes	1, 2	19	0
Explaining characteristics of plane shapes	3, 4, 5	19	0
Measuring area using non-standard units	6, 7, 8	15	4
Measuring area using standard units	9, 10	3	16

The results of the cognitive diagnostic assessment in Table 5 show that all students answered questions 1 to 5 correctly. However, four students were unable to answer questions 6, 7, and 8, indicating that some students had not yet mastered the prerequisite skill of measuring area using non-standard units. Therefore, in the first meeting, the lesson began with a review of this material, and homework was assigned to the four students to strengthen their understanding. For questions 9 and 10, most students gave incorrect answers. This is understandable, as the topic had not yet been taught in school and would be introduced through the PBL model. Interview results revealed that the three students who answered questions 9 and 10 correctly had learned the material independently with their parents or through tutoring.

The example of a student's work on the non-cognitive diagnostic assessment is presented in Fig. 3.



Bagian A. Learning Preference

1. Saya lebih mudah memahami pelajaran ketika belajar sendirian.
   - ☐ Sangat Tidak Setuju
   - ☐ Tidak Setuju
   - ☑ Ragu-ragu
   - ☐ Setuju
   - ☐ Sangat Setuju
2. Saya lebih senang berdiskusi dengan teman untuk memahami materi.
   - ☐ Sangat Tidak Setuju
   - ☐ Tidak Setuju
   - ☐ Ragu-ragu
   - ☑ Setuju
   - ☐ Sangat Setuju
3. Saya lebih fokus ketika belajar tanpa ada orang lain.
   - ☐ Sangat Tidak Setuju
   - ☐ Tidak Setuju
   - ☑ Ragu-ragu
   - ☐ Setuju
   - ☐ Sangat Setuju
4. Saya lebih semangat belajar saat ada kerja kelompok.
   - ☐ Sangat Tidak Setuju
   - ☐ Tidak Setuju
   - ☐ Ragu-ragu
   - ☑ Setuju
   - ☐ Sangat Setuju

The translation:
Part A. Learning Preference

1. I find it easier to understand lessons when studying alone.
   - Strongly Disagree
   - Disagree
   - Neutral
   - Agree
   - Strongly Agree
2. I prefer discussing with friends to understand the material.
   - Strongly Disagree
   - Disagree
   - Neutral
   - Agree
   - Strongly Agree
3. I am more focused when studying without others around.
   - Strongly Disagree
   - Disagree
   - Neutral
   - Agree
   - Strongly Agree
4. I am more motivated to study during group work.
   - Strongly Disagree
   - Disagree
   - Neutral
   - Agree
   - Strongly Agree

Fig. 3. Student's response of non-cognitive diagnostic assessment

Fig. 3 shows the student's response in the non-cognitive diagnostic assessment on the learning preference aspect. The student's work indicates a tendency to prefer group learning. The results of the non-cognitive diagnostic assessment on students' learning preferences are presented in Table 6.

Table 6. Result of non-cognitive diagnostic assessment

Aspect	Type	Number of students
Learning preference	Individual	8
	Group	11
Learning style	Visual	7
	Auditory	4
	Kinesthetic	8

Based on the non-cognitive diagnostic assessment data in Table 6, most students preferred group-based learning, although some showed a preference for individual learning. The learning style analysis indicated that kinesthetic learning was the most dominant, followed by visual and auditory styles. This information served as the basis for designing learning tools using the TaRL approach and the PBL model. The developed tools were designed not to favor

any single learning style or preference. They were intended to accommodate all existing learning preferences and styles.

B. Level of Critical Thinking Skills

Students' critical thinking skills in both classes were measured using pretest and posttest essay items. The results from the experimental and control classes are presented in Table 7, Table 8, and Table 9.

Table 7. Mean score of pre test & post test

Group	Pre test	Description	Post test	Description
Experimental	53.33	Moderate	80.53	Very high
Control	52.00	Moderate	70.33	High

Table 8. Mean score of the experimental class for each aspect

Aspect	Pre test		Post test	
	Mean	Description	Mean	Description
Interpretation	55.33	Moderate	89.46	Very high
Analyze	50.67	Moderate	73.33	High
Inferential	54.25	Moderate	81.53	Very high
Evaluation	50.67	Moderate	72.67	High
Eksplanation	55.82	Moderate	85.67	Very high

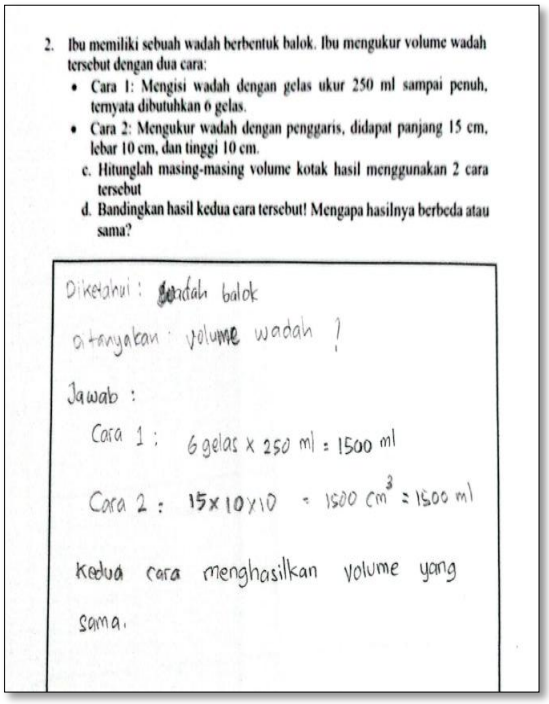
Table 9. Mean score of the control class for each aspect

Aspect	Pre test		Post test	
	Mean	Description	Mean	Description
Interpretation	53.47	Moderate	72.46	High
Analyze	51.67	Moderate	70.33	High
Inferential	51.33	Moderate	69.23	High
Evaluation	50.89	Moderate	69.46	High
Eksplanation	52.67	Moderate	70.19	High

Based on Table 7, the average score of mathematical critical thinking increased in both groups. The improvement occurred due to the implementation of the PBL model. The experimental class had a pretest average score of 53.33, classified as moderate, and a posttest average of 80.53, classified as very high. In the control class, the average pretest score was 52.00 (moderate), and the posttest average was 70.33 (high). Although both groups showed improvement, the experimental class achieved a higher score and category. This difference was due to the use of the TaRL approach in experimental class. The TaRL-based learning ensured students had mastered the prerequisite material before learning new content. This helped students learn more effectively.

Tables 8 and 9 show the mean scores of the experimental and control classes across five aspects. In the experimental class, all aspects improved significantly from a moderate to high or very high level after the intervention. The most notable increase occurred in the

interpretation and explanation aspects, rising from 55.33 to 89.46 and from 55.82 to 85.67, respectively. In contrast, the control class also showed improvement but to a lesser extent, with all post-test means remaining at the high level. These results indicate that the treatment given to the experimental class had a stronger positive effect on students' critical thinking skills. One example of a student's post-test work is shown in Fig. 4.



Question:
2. A mother has a container in the shape of a cuboid. She measures the volume of the container in two ways:

- **Method 1:** Filling the container with a 250 ml measuring cup until full, it turns out she needs 6 cups.
- **Method 2:** Measuring the container with a ruler, she finds the length is 15 cm, the width is 10 cm, and the height is 10 cm.
  a. Calculate the volume of the box-shaped container using these two methods.
  b. Compare the results of both methods. Why are the results different or the same?

Student's Answer:
Known: cuboid
Asked: volume of the container?
Answer:

- **Method 1:** 6 cups × 250 ml = 1500 ml
- **Method 2:** 15 × 10 × 10 = 1500 cm^3 = 1500 ml

Both methods produce the same volume.

Fig. 4. Student's response of critical thinking test

The result demonstrates the student's critical thinking ability, as they were able to solve the problem using two different methods. From both approaches, the student was able to explain that the results were the same even though the volume units were different. This outcome also indicates that the student has already acquired the basic ability to convert units from cm^3 to ml.

C. The Combination of TaRL and PBL to Improve Critical Thinking Skills

A hypothesis test was conducted to determine whether the improvement in scores after the learning intervention was statistically significant. Prior to the hypothesis test, a normality test was performed. The normality test using the Shapiro–Wilk method is presented in Table 10.

Table 10. Result of normality test

Group	test	Sig.
Experimental	Pre test	0.163
	Post test	0.137
Control	Pre test	0.051
	Post test	0.126

Based on Table 10, the normality test results indicate that the pretest and posttest data in both groups are normally distributed, as the significance values are greater than 0.05. Therefore, an independent t-test was conducted to examine whether there was a difference in pretest scores between the two groups. The results are shown in Table 11.

Table 11. Independent T-test Output: Pretest Scores of Critical Thinking Skills

		Levene's test for equality of variances		t-test for equality of means			
		F	Sig	t	Df	Sig.2-tailed	Mean Differences
Critical Thinking – Pre test	Equal variances assumed	.238	.627	.892	32	.237	11.600
	Equal variances not assumed			.892		.242	11.600

Table 11 shows that the Sig. 2-tailed value is greater than 0.05. This indicates no significant difference in the average pretest scores of critical thinking skills between the two groups. It means the students in both groups had similar initial abilities before the learning intervention.

Table 12. Independent t-test Output: Posttest Scores of Critical Thinking Skills

		Levene's test for equality of variances		t-test for equality of means			
		F	Sig	t	Df	Sig.2-tailed	Mean Differences
Critical Thinking – Post test	Equal variances assumed	.001	.981	2.462	32	.019	5.429
	Equal variances not assumed			2.462		.021	5.429

Table 12 shows that the Sig. 2-tailed value is 0.019, which is less than 0.05. This indicates a significant difference in the average posttest scores of critical thinking skills between the two groups. Since a significant difference was identified, a one-tailed t-test was performed. The results are shown in Table 13.

Table 13. One-tailed t-test Output: Posttest Scores of Critical Thinking Skills

Group	Mean	Variance	t - calculated	t-table
Experimental	80.53	35.82	2.462	1.693
Control	70.33	30.24		

Table 13 shows that the t-calculated value is 2.462, while the t-table value based on the t-distribution at a significance level of 0.05 is 1.693. Since the t-calculated value is greater than the t-table value, it can be concluded that the average posttest score of critical thinking skills in the experimental group is significantly higher than that of the control group. This result is

likely due to the experimental group using a combination of the TaRL approach and the PBL model, whereas the control group only implemented PBL.

Next, a paired t-test was conducted to measure the impact of combining TaRL and PBL on mathematical critical thinking skills in the experimental class. The analysis showed that the t-test for critical thinking skills with 32 degrees of freedom yielded a significant value of 0.004. The hypothesis test indicated that the significance value of 0.004 (< 0.05) demonstrates a significant difference in the average critical thinking scores between the pre-test and post-test. This confirms the positive influence of the TaRL and PBL implementation on students' mathematical critical thinking skills. This finding is consistent with previous studies that emphasized the significant improvement in students' numeracy resulting from the integration of PBL and TaRL [28]. Numeracy is a form of critical thinking that requires more than just the ability to manipulate numbers [29].

The magnitude of the effect of combining TaRL and PBL on mathematical critical thinking skills was further measured using Cohen's *d* effect size. The analysis produced an effect size of 0.846 on a 0–1 scale, which falls within the large category. This calculation provides additional information about the strength of TaRL and PBL's impact on students' mathematical thinking skills. It shows that the integration of these two approaches not only achieves statistical significance but also exerts a strong and substantial influence on enhancing students' mathematical critical thinking skills.

TaRL's approach, which adjusts instruction to students' actual competence levels, effectively accelerates mastery of prerequisite skills needed for new material [30]. In this study, using cognitive diagnostic assessment before intervention enabled the teacher to identify students' prerequisite weaknesses and design appropriate instruction [31]. Employing non-cognitive diagnostic assessment also supported differentiated lesson planning. The findings show that students had diverse learning preferences, with most favoring kinesthetic and collaborative learning. The teacher used this information to design learning experiences that adapt to individual and group learning styles. This approach enhanced learning experiences, increased engagement, promoted personalized learning, and strengthened long-term knowledge retention and skill mastery [32].

The Problem-Based Learning (PBL) model uses a structured instructional syntax designed to stimulate higher-order thinking skills, including mathematical critical thinking [33]–[35]. This syntax involves five main stages that unfold sequentially. It begins with problem orientation, where students are introduced to authentic issues, followed by student organization for learning, which encourages collaborative planning and engagement. The process then continues with guided individual and group investigations, leading to the development and presentation of student work. Finally, it culminates in the analysis and

evaluation of the problem-solving process [36]. Each stage plays a key role in developing aspects of critical thinking: interpretation, analysis, inference, evaluation, and explanation [37].

In the problem orientation stage, the teacher presents challenging, open-ended contextual problems. In this study, the teacher provided ill-structured problems on measuring the area of irregular plane shapes. This stage stimulated students to interpret initial information, identify relevant data, and understand the mathematical context of the problem [38], [39]. It triggered early critical thinking skills through information gathering and data identification. The organization stage promoted collaboration, communication, and active discussion. During this process, students began analytical thinking by breaking down problems, determining solution strategies, and connecting relevant mathematical concepts [40]–[42].

During the investigation stage, students explored data, tested hypotheses, and sought solutions using mathematical approaches. These activities closely engaged inference and evaluation, as students reasoned and developed deep understanding to solve problems [43]. The presentation stage allowed students to explain and communicate their thought processes orally or in writing [44]. Explanation skills became particularly evident, as students had to justify their solution steps to peers or the teacher.

In the analysis and evaluation stage, students critically reconsidered the effectiveness of strategies, the accuracy of solutions, and potential alternative approaches. This stage reinforced the metacognitive dimension of critical thinking, evaluating one's thinking processes and improving understanding quality [45], [46].

Conclusion

The purpose of this study is to examine whether the integrated approach of Teaching at the Right Level (TaRL) and Problem-Based Learning (PBL) can improve elementary students' mathematical critical thinking skills. Based on the findings in the results and discussion section, this objective was achieved. The combination of TaRL and PBL significantly improved students' critical thinking, particularly in the indicators of interpretation, inference, and explanation. The t-test results showed a significant difference between pretest and posttest scores in the experimental group, with a large effect size, confirming the effectiveness of the TaRL–PBL approach. The use of cognitive and non-cognitive diagnostic assessments and student grouping based on learning levels aligns with the core principles of TaRL and supports effective differentiated instruction. When combined with the PBL model, students are not only able to solve problems procedurally but also able to explain their thinking processes logically and deeply.

Limitation and Suggestions (optional)

These findings contribute significantly to the development of more adaptive and challenging mathematics learning designs in elementary education. This study is limited to the variables of TaRL, PBL, and mathematical critical thinking. Future research may explore combinations with other variables such as creative thinking, mathematical connections, or other subjects.

Conflict of Interest

The authors state no conflict of interest. They declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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