Applying Robust Fuzzy-stochastic Programming Model in Multi-objective Optimization for Flow-shop Scheduling with Parallel Machines: a Real-life Case Study in an Engine Component Industrial

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***Abstract*—Resources scheduling, jobs sequencing, and resources assignment are considered the brain activities that play an extremely important role to the business, such as flow-shop scheduling. Additionally, the internal manufacturing environment and the current market are drastically changing, in which attracted significant attention in scheduling under uncertainties. This research aims to investigate the multi-objective flow-shop scheduling with parallel machines under machine breakdown and operational uncertainty. A novel robust fuzzy stochastic programming (RFSP) model is presented to address the unexpected operational disruptions and the inherent uncertainty of processing time. The objective is to optimize the cost of late delivery and the efficiency of manufacturing evaluated by overall equipment effectiveness (OEE) whilst considering the CO2 emissions produced. Eventually, a proposed algorithm is applied with a real-life scheduling data from a German factory that faces a practical situations in machine breakdowns and operational risks with unrelated parallel machines. Thus, this study intends to provide the computational results and useful information for multiple decision processes.**

***Keywords*—Flow-shop Scheduling, Machine Breakdown, Mixed-integer Programming, Parallel Processing, Robust Fuzzy Stochastic Programming, Sequencing, Uncertainty.**

I. INTRODUCTION

Flow-shop scheduling with parallel machines (FSSPM) is widely applied in practical industrial systems. Many manufacturing architectures include an ordinary flow shop, which consists of a multi-stage production process requiring all items to proceed through several stages in the same sequence. In a traditional flow shop, each stage consists of a single machine that is narrowly capable of carrying out a single job at a time. It represents a greater realistic to consider that, at every workstation, some identical machines may function in parallel, known as flow shops with parallel machines. This very concept of scheduling environment, which is rather prevalent, has been observed in industries such as semiconductors, engine components, electronics manufacturing, and petrochemical production [1]. However, a flow shop with several parallel machines further complicated the scheduling challenges, as the work processing order at each workstation can be extremely fluctuate.

Furthermore, the typically assumptions of scheduling challenges in literature are deterministic. In contrast to real-life scheduling applications, multiple non-deterministic and uncertain factors influence the performance metric. In recent years, there has been an explosion of interest in investigating scheduling problems with non-deterministic or unknown parameters. In this circumstances, Ref. [2] explored the issue of unpredictable demand. They created various criteria to assess issue resilience and used certain extreme demand bounds as scenarios to improve batch plant scheduling model performance under assumed unknown parameters. Ref. [3] devised a robust optimization strategy in their multi-period programming model to account for unknown processing time, demand, and price. The selected robust optimization approach was quite conservative and deemed to be of worth-case value for these parameters, hence they assumed bounds for unknown values. Ref. [4] investigated difficulties with unclear data. For some scenarios involving unknown parameters and a known probability distribution, they employed a robust optimization strategy. Their robust optimization technique produced a deterministic robust equivalent with a few extra variables and restrictions. Ref. [5] investigated insights into robust and consistent flow shop scheduling in the context of unexpected novel job arrivals and unknown processing timeframes. They utilized a scenario-based robust-stochastic optimization technique. For an overview of the prior research on robust scheduling issues, review the survey publications by Ref. [6] [7].

Despite the aforementioned enhancements have render the problem practical, taking into account the uncertainties inherent in the production process brings the problem closer to the real world. Unexpected events, including machine breakdowns, might occur in the workplace. Such circumstances may lead the optimal schedule to become insufficient, making it unfeasible in new conditions. Additionally, certain new information, such as the new job's due date and setup time, may be updated during the process. Accordingly, providing revised schedules that can address the uncertainties caused by unexpected disruptions such as stochastic break down scenario along with fuzzy uncertain processing time [8]. This research has successfully combined the stochastic scenario-based methods and fuzzy numbers to introduce a novel robust fuzzy stochastic programming

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(RFSP) model. However, the framework of the presented problem investigates the dual-resource constrained flexible job-shop scheduling and have a limit in implement the proposed model in the real-world case study.

Thus, in order to generate effective, dependable, and legitimate schedules for FSSPM processes that is sufficient in the real world, it is critical that systematic methodologies be developed to handle the challenges of FSSPM under uncertainty. Nonetheless, the practical issue of non-deterministic or uncertain parameters in FSSPM scheduling has not been addressed so far and absenced of real-life case study to validate the proposed robust model. To overcome all the above mentioned deficiencies and address this gap, this study investigates the multi-objective FSSPM problem with machine breakdowns and fuzzy processing time by applying a novel robust fuzzy stochastic programming (RFSP) model. Additionally, the suggested robust model is implemented and analyzed with a real-world data from a German factory that faces a relevant practical situations.

The structure of this article is organized as follows. Section 2 provides a comprehensive review of relevant research on flow-shop scheduling with parallel machines, highlighting the objectives pursued in previous studies and identifying the existing research gap. Furthermore, in Section 3, we present a detailed description of the manufacturing architecture considered in our model framework based on a real-world case study and propose a FSSPM optimization model for the problem. To cope with these challenges, Section 5 introduces the robust fuzzy stochastic programming (RFSP) technique, which effectively handles machine breakdowns and uncertainty in setup time and due dates. This technique provides a robust and reliable approach to scheduling in the face of uncertainties. In Section 6, we present a real-world case study conducted at a German plant to evaluate the effectiveness of the RFSP model proposed and analyze sensitivity in our research. Finally, this study concludes with suggestions for further research. We identify potential areas of investigation that can build upon our findings and contribute to the advancement of flow-shop scheduling with parallel machines.

II. Literature Revew

Over the past few decades, flow shop scheduling has been thoroughly investigated and published in the literature; however, practically all of these studies examine primarily single machine and multiple stage flow shops. To investigate the objective scopes of recent research about flow-shop scheduling under uncertainty, this study has randomly chosen a considerable number of studies with the keyword ‘flow-shop scheduling’ and ‘under uncertainty’ in Google Scholar. The findings of a brief novelty for 20 selected literature among the last ten years is presented in Table I. This table effectively showcases the various approaches employed to address uncertainty, the specific objective functions investigated, and whether the study framework is based on a case study or not.

There are multiple approaches to address the uncertainty in scheduling problem including probabilistic, non-probabilistic, scenario-based and inverse uncertainty. Depend on the framework, the uncertain factors considered by various research can be exceedingly diverse. One of the most common uncertain parameters have been investigated is processing time [12, 14, 17, 20, 22, 24, 28]. Due date [12, 24, 27]. A robust fuzzy optimization models solving a flow shop scheduling problem with sequence dependent setup times has proposed by Gholami-Zanjani *et al.* [23]. Moreover, Marichelvam and Geetha [13] have developed an energy-efficient flow shop scheduling considering to rush orders, cancelled orders and the overall workers availability to minimize the overall energy cost. Furthermore, an uncertain assembling times of the products are assumed to follow a normal distribution in order to address a two-stage assembly flow shop scheduling problem [11].

Coming to the scenario-based approach, the machine breakdown and preventive maintenaince have received widespread application throughout 20 literature reviews [9, 15, 17, 18, 21, 25, 26]. Levorato *et al.* [10] proposed a polynomial time technique to handle uncertainty and optimize the makespan, while also providing a case study. Moreover, due date uncertainty has also been considered in several studies [12, 24, 27]. These studies investigate how uncertain due dates affect scheduling decisions and propose approaches to handle such uncertainties effectively.

The primary objective function considered in numerous studies is the makespan, aiming to minimize the total completion time of all jobs [9, 10, 11, 16, 19, 20, 22, 28]. Other objectives, such as minimizing energy costs [13], optimizing the overall equipment effectiveness (OEE) [15], and considering CO_2 emissions (noted in the abstract), have also been explored. Several studies have incorporated real-world case studies to validate their proposed models and algorithms [10, 13, 15, 16, 17, 18, 19, 20, 23, 26] . These case studies provide practical insights into the application of flow-shop scheduling under uncertain conditions.

In contrast to prior research endeavors, this study undertakes a comprehensive examination of the internal and external uncertainties that are inherently associated with the FSSPM domain. Furthermore, it devotes attention to scenario-based machine breakdowns and preventive maintenance practices, which have often been overlooked in existing scholarly investigations. By incorporating these crucial aspects, this study achieves a more realistic and holistic understanding of the complexities encountered in actual manufacturing environments. Moreover, the integration of a real-life case study enhances the practical relevance and credibility of the findings, thereby positioning this study as a significant advancement within the realm of FSSPM research.

III. Fsspm Problem

A. *FSSPM problem statement*

Flow-shop scheduling problems exist in multi-product batch plants where tasks of production orders run the same processing units in the same sequence (Birewar & Grossmann, 1989). Furthermore, this type of scheduling is defined as FSSPM problem when the manufacturing environment have parallel machines in several workstations as an integration of two traditional scheduling issues (flow-shop and parallel machines).

It can be seen in the Figure 1 presenting the general framework of FSSPM problem. Every jobs requires to be processed through all workstations and go through the identical procedure of exactly one machine each time, i.e. a permutation flow-shop plant. Given each job has its due date DD_ialong with unrelated set up time ST_j and processing time P_{ij} depending on different machine in the shop. The optimization model will be proposed in the next sub-section with the purpose of scheduling when and what jobs will be

TABLE I. A BRIEF REVIEW OF RELATED LITERATURE

Ref	Demand	Scenario-based approach	Operational Uncertainty	Objective Measure	Case study
[9]		Machine breakdown		Makespan	
[10]		Polynomial time		Makespan	x
[11]			Assembling times	Makespan	
[12]	Due date		Processing time	Total tardiness, makespan	
[13]			Rush orders, cancelled orders, overall workers availability factor	Total energy cost	x
[14]			Processing time		
[15]		Breakdown and planned preventive maintenance		Makespan, delays	x
[16]	Demand volume			Makespan	x
[17]		Duration of maintenance activities	Processing time	Total expected cost associated with lateness penalties and maintenance resources	x
[18]		Stochastic processing time		Makespan, energy consumption	x
[19]	Demand volume			Makespan	x
[20]			Processing time	Makespan	x
[21]		Breakdown and planned preventive maintenance			
[22]			Processing time	Makespan	
[23]			Processing time and Setup time	Weighted mean completion time	x
[24]	Due date		Processing time	Energy consumption, tardiness	
[25]		Machine breakdowns		Stability and robustness	
[26]		Inherently unexpected events		Total weighted tardiness	x
[27]	Due date			Makespan, total tardiness	
[28]			Processing time	Makespan	
This study	Due date	Machine breakdowns and operational risks	Setup time	Cost of tardiness and storage, OEE, CO_2 emissions	x

implemented on which machine to qualify the multi objectives presented in Section V.

B. *Proposed FSSPM optimization model*

Following the general problem statement presented above, the related indices, parameters, variables, and the mixed integer optimization model is proposed in this sub-section.

Indices

i all jobs to be scheduled
{1, 2, 3, …, I}

ACT_i all jobs to be scheduled at the present time

j all entities in the shop (machines and workstations)
{W1, M.a,…, M.d, W2, M.e, M.f, W3, M.g,…, M.i, W4, M.j}

WS_j all workstations in the shop
{W1, W2, W3, W4}

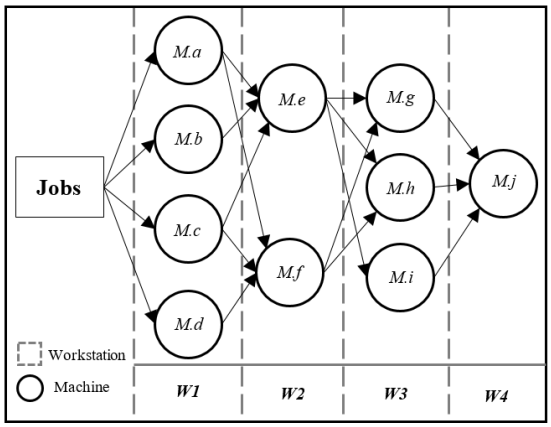


Fig. 1. FSSPM framework

MA_j all machines in the shop
{M.a, M.b, M.c,…, M.i, M.j}

SM_j all single machines in the shop
{M.j}

PA_j all sets of parallel machines for each workstations
$\{W1_j, W2_j, W3_j, W4_j\}$

ξ all machine breakdown scenarios
{1, 2, 3, …, Ξ}

Input parameters

$\widetilde{DD}_i$ uncertain due dates for job *i*

Q_{ij} batch size for job *i* to be processed in entity *j*

P_{ij} processing time for a batch size of job *i* in entity *j*

$\widetilde{ST}_j$ uncertain setup time for machine MA_j to be processed

L a very big number

$$SEC_{i,j,jj} = \begin{cases} 1, \text{when job } i \text{ is processed on machine } jj \text{ immediately after being processed on machine } j \\ 0, otherwise \end{cases}$$

F number of workstations needed to finish to produce the job (or constant for the flow–shop scenario)

PC_i late order penalty cost for job *i*

SC_i unitary storage cost per unit of time for job *i*

EC_j CO_2 emissions rate for an hour of process at entity j

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FC CO_2 emissions for each late job sequenced
NC CO_2 emissions for each normal job sequenced
R_j repair time of machine *j* for each number of breakdowns
ρ worst-case importunacy coefficient
π_ξ Probability of occurrence of breakdown scenario ξ
$B_{j,\xi}$ Number of breakdowns of machine *j* under scenario ξ per time unit
α fuzzy credibility level ($\alpha \geq 0.5$)

Decision Variables

x_{ij} starting time for job i at machine j ($x_{i\,j} \geq 0$)

$$w_{ij} = \begin{cases} 1, when\ job\ i\ is\ processed\ on\ machine\ j \\ 0, otherwise \end{cases}$$

$$y_{i,ii,j} = \begin{cases} 1, when\ job\ i\ is\ processed\ on\ machine\ j\ before\ job\ ii \\ 0, otherwise \end{cases}$$

Dependent Variables

xx_{ij} a dummy variable with the starting time for job *i* at machine *j* ($xx_{i\,j} \geq 0$)
e_i earliness of job *i* ($e_i \geq 0$)
t_i tardiness of job *i* ($t_i \geq 0$)
c_i completion time of job *i* at entity *j*

$$d_i = \begin{cases} 1, if\ job\ i\ is\ delivered\ after\ its\ due\ date \\ 0, otherwise \end{cases}$$

$r_{j,\xi}$ Number of breakdowns of machine *j* under scenario ξ ($r_{j,\xi} \geq 0$)
$M(f(x))$ Mean of the objective value under all of the scenarios
$W(f(x))$ Worst-case of the objective value under all of the scenarios

The goal function for the production order is displayed in the mathematical model for the problem that the corporation suggested to solve below. This paradigm may also be applied to other objective functions with little modifications, as will be demonstrated later.

$$Min\ f_{(x)}$$

subject to.

$$c_{i,j} = x_{i,j} + \left(P_{i,j}Q_{i,j} + \widetilde{ST_j}\right)w_{i,j}, \quad \forall i \in ACT_i \tag{1}$$

$$c_{i,j} \leq x_{i,jj}, \quad \forall i,j \in ACT_i, SEC_{i,j,jj} = 1 \tag{2}$$

$$c_{i,j} \leq x_{ii,j} + L\left(1 - y_{i,ii,j}\right), \quad \forall i,ii \in ACT_i, j \in MA_j, i \neq ii \tag{3}$$

$$c_{ii,j} \leq x_{i,j} + Ly_{i,ii,j}, \quad \forall i,ii \in ACT_i, j \in SM_j, i \neq ii \tag{4}$$

$$c_{ii,j} \leq x_{i,j} + Ly_{i,ii,j} + L * (1 - w_{i,j}), \quad \forall i,ii \in ACT_i, j \in PA_j, i \neq ii \tag{5}$$

$$xx_{ij} = x_{ij}, \quad \forall i \in ACT_i, j \in PA_j \tag{6}$$

$$xx_{ij} - Lw_{ij} \leq 0, \quad \forall i \in ACT_i, j \in PA_j \tag{7}$$

$$xx_{i,j} - x_{i,c} \leq 0, \quad \forall i \in ACT_i, j \in PA_j \tag{8}$$

$$x_{i,c} - xx_{ij} + Lw_{ij} \leq L, \quad \forall i \in ACT_i, j \in PA_j \tag{9}$$

$$\sum_{j \in PA_j} w_{ij} = 1, \quad \forall i \in ACT_i \tag{10}$$

$$\sum_{i \in I}\sum_{j \in J} w_{ij} = F * I \tag{11}$$

$$e_i \geq \widetilde{DD}_i - c_{i,J}, \quad \forall i \in ACT_i \tag{12}$$

$$t_i \geq c_{i,J} - \widetilde{DD}_i, \quad \forall i \in ACT_i \tag{13}$$

$$t_i \leq Ld_i, \quad \forall i \in ACT_i \tag{14}$$

Eq. (1) calculates the completion time of each job. Constraint (2) responsibles for the starting time of the job on the next machine is always after the completion time of that job on the current machine. Constraints (3), (4) and (5) ensures that when a machine *j* is processing a part *i*, no other part can be scheduled for processing on the same machine at the same time. Their function is to determine the order in which jobs enter each machine. Big–L relaxation allows enforcement of the logical constraint: either job *i* enters machine *j* before job *ii*, or job *ii* enters machine j before job *i*. Equations (6) and (7) create a dummy variable so as not to lose that information; when the resource is not used ($w_{ij} = 0$), the value of this variable is set to zero. Equations (8) and (9) assign the value of the starting time at machine *j* (xx_{ij}) as the starting time at the workstation. Equations (10) ensure that when more than one resource is available for a given job, every job is processed in only one machine. When machines work in parallel, Eq. (11) ensures that no part skips any job and that all parts follow the corresponding job sequence, with *I* is the total jobs. Eq. (12) and (13) define a job's delivery time as early or late, respectively. Eq. (14) is the activation constraint of variable d_i for delayed jobs.

IV. RFSP Optimization Model

A. Stochastic approach

Scenario-based stochastic programming (SSP) is the typical technique for scheduling, planning, and designing issues with interruption risk [29, 30, 31]. The SSP method classifies decision factors into two categories: scenario-independent and scenario-dependent. The independent variables have the exact identical value in all situations. It includes whilst the dependent variables can be adapted for every scenario. The SSP technique responds to the independent factors; consequently, modifying the dependent variables keeps the issue doable and near to optimum under every possible circumstance. The SSP's objective function is to maximize the system's average performance across all circumstances.

Generally, the SSP approach is likely to be described in Eq. (15).

$$\begin{cases} Min \sum_{\xi \in \Xi} \pi_\xi z_\xi \\ z_\xi = a_\xi^{\mathrm{T}} x_\xi + b_\xi^{\mathrm{T}} y, \quad \forall \xi \in \Xi \\ A_\xi x_\xi + B_\xi y \geq c_\xi, \quad \forall \xi \in \Xi \\ My = q \\ y \in Y,\ x_\xi \geq 0 \end{cases} \tag{15}$$

where z_ξ presents the objective function under scenario $\xi \in \Xi$, x_ξ is the vector comprising scenario-dependent variables, while, the scenario-independent variable vector is denoted by y. Sets of $(a_\xi^T, b_\xi^T, A_\xi, B_\xi, c_\xi)$ characterize the parameters, (M, q) present the certain parameters, and π_ξ is the probability for each scenario.

B. Fuzzy approach

Otherwise, robust fuzzy/possibilistic programming (RPP) is used for data from optimization problems with inherent uncertainty that may be represented as a fuzzy number. RPP is employed in fuzzy optimization considering data is given as triangular fuzzy numbers (TFNs) (presents in Fig. 2). This approach is similar to the mean value of a fuzzy number established by Dubois and Prade [32] in the context of Dempster/Shafer theory. The framework defines a fuzzy number $\widetilde{\mathbb{F}}$ as a collection of potential probability distributions. The mean value of $\widetilde{\mathbb{F}}$ is the set of mean values determined according to all of these probability distributions. In contrast with SSP, the RPP approach is implemented to deal with the inherent uncertainty and operational risk when uncertain parameters are treated as trapezoidal or triangular fuzzy numbers [33].

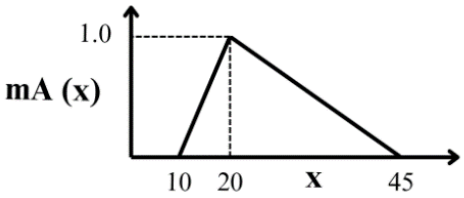


Fig. 2. The function of a TrFN number

Constraint satisfaction is measured using lower mean value X_* and the upper mean value X^*. The distribution functions $F_*(x)$ and $F^*(x)$ is utilized as follows in Eq. (16)-(19).

$$F_*(x) = inf\{1 - \mu_{\widetilde{\mathbb{F}}}(t) | t \geq x\} \quad (16)$$

$$F^*(x) = sup\{\mu_{\widetilde{\mathbb{F}}}(t) | t \leq x\} \quad (17)$$

$$X_* = \int_{-\infty}^{+\infty} x dF_*(x) \quad (18)$$

$$X^* = \int_{-\infty}^{+\infty} x dF^*(x) \quad (19)$$

Hence, the interval $[X_*, X^*]$ represents the mean value of the fuzzy number $\widetilde{\mathbb{F}}$. Assuming that any value inside the interval $[X_*, X^*]$ has an equal probability, the midpoint of this interval would be the obvious option for a single real integer. According to [34], the area compensation procedure yields a real number that, in the context of the Dempster/Shafer theory, where both sides of the possibility distribution induce a probability measure, is the midpoint of the interval corresponding to the mean value of a fuzzy number. Consequently, we have (20).

$$AC(\widetilde{\mathbb{F}}) = 0.5 \int_0^1 (x_\alpha^L + x_\alpha^R) d\alpha = 0.5(X_* + X^*) \quad (20)$$

C. Robust fuzzy-stochastic programming (RFSP) approach

The RFSP methodology is a contemporary optimization utilizing the uncertainty technique to handle planning under simultaneous disruption/breakdown scenarios and operational unpredictability risk [35]. RFSP basically integrates conventional scenario-based stochastic programming (SSP) and robust fuzzy/possibilistic programming (RPP).

However, classical possibilistic programming contains limitations in ensuring that the decision-maker determines the value of α subjectively and adjusts it passively or interactively, which may not be optimal. To address this issue, Pishvaee *et al.* [36] introduced a robust possibilistic programming (RPP) model that proactively and objectively determines the ideal amount of α.

In the SSP model (15), c_ξ is specified as a TFN uncertain parameter, which can be scenario-independent. If we employ the RPP model for handling fuzziness and the SSP approach presented in Formula (15) to manage all possible scenarios. Moreover, the current RFSP model has a significant drawback. The approach focuses solely on the objective function's expected value and ignores system performance in worst-case scenarios. To address this, the goal function is expanded by include the worst-case performance under all conditions.

The RFSP model adding the worst-case performance under all scenarios can be written as follows:

$$\begin{cases} Min \sum_{\xi \in \Xi} \pi_\xi z_\xi + p \max_{\xi \in \Xi}(z_\xi) \\ + \left(\frac{SS}{3}\right)\left(\frac{1}{2}\right) \sum_{\xi \in \Xi} \pi_\xi \sum_{i \in I} \sum_{\alpha \in \mathbb{A}} C_\alpha (z_\xi^{\alpha,L}(x) + z_\xi^{\alpha,R}(x)) \\ z_\xi = a_\xi^T x_\xi + b_\xi^T y, \quad \forall \xi \in \Xi \\ A_\xi x_\xi + B_\xi y \geq c_\xi^{\alpha,e}, \quad \forall \xi \in \Xi \\ My = q \\ y \in Y, \ x_\xi \geq 0 \\ \alpha \geq 0.5 \end{cases} \quad (21)$$

where the decision-maker determine the worst-case significance coefficient, denoted by $\rho \geq 0$ and α and e respectively are the fuzzy sets represents for the fuzzy credibility level and the indication of the left and right end-points used in the interval-arithmetic. The objective area-compensation integral is discretely approximated in Eq. (21) where the C_α represent as the Simpson coefficients utilized in the integral approximation. In the next subsection, we apply the aforesaid RFSP model to develop the FSSPM problem.

D. RFSP mathematical model for FSSPM problem

Within production systems, an abundance of data derived from business information systems typically exists, presenting an opportunity to leverage these data for uncertainty modeling. It is essential to distinguish between internal parameters pertaining to the production system, such as processing times, setup times, scrap rates, and machine availability, and external parameters primarily characterized by fluctuating demand. The utilization of this data allows for an enhanced understanding and representation of uncertainties present in the production environment.

In this case, set-up times are subject to internal manufacturing fluctuations, such as manual processes performed by operators, the occurrence of rush orders or cancelled orders, the availability of workers, and the time required for material movement. By modeling set-up times as triangular fuzzy numbers, the inherent variability and uncertainty associated with these factors can be effectively captured. Similarly, due dates are influenced by fluctuations in the supply chain, including changes in customer demands and resources. Incorporating these uncertainties as triangular

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fuzzy numbers enables a more realistic representation of the dynamic nature of customer demand.

Additionally, the consideration of machine breakdowns and preventive matainance as scenario-based uncertainty acknowledges the potential disruptions that can occur in real-world manufacturing environments. By incorporating these scenarios, the scheduling model can account for the impact of machine breakdowns on the production process and develop robust scheduling strategies to mitigate their effects.

Therefore, the robust model that takes uncertain set-up time, due date and machine breakdown into account is proposed in this section.

subject to.

$$M\left(e_i^{\xi,\alpha,e}SC_i + t_i^{\xi,\alpha,e}PC_i\right) = \sum_{\xi\in\Xi}\pi_\xi \sum_{e\epsilon E}\left(\frac{1}{E}\right)\sum_{\alpha\in\mathbb{A}}\sum_{i\in ACT_i}\left(\frac{1}{\mathbb{A}}\right)(e_i^{\xi,\alpha,e}SC_i + t_i^{\xi,\alpha,e}PC_i) \quad (22)$$

$$W\left(e_i^{\xi,\alpha,e}SC_i + t_i^{\xi,\alpha,e}PC_i\right) = \max_{\xi\in\Xi}\left\{\pi_\xi \sum_{i\in ACT_i}(e_i^{\xi,\alpha,e}SC_i + t_i^{\xi,\alpha,e}PC_i)\right\}, \forall\alpha \in \mathbb{A}, e\epsilon E \quad (23)$$

$$r_{j,\xi} \geq B_{j,\xi}.(P_{i,j}w_{i,j}^{\xi}Q_{i,j}), \forall j \in MA_i, \xi\in\Xi, \alpha\in \mathbb{A}, e\epsilon E \quad (24)$$

$$c_{i,j}^{\xi,\alpha,e} = x_{i,j}^{\xi} + (P_{i,j}Q_{i,j} + ST_j^{\alpha,e})w_{i,j}^{\xi} + R_j r_{j,\xi}, \forall i\in ACT_i\ j\in WS_i, \xi\in\Xi, \alpha\in\mathbb{A}, e\epsilon E \quad (25)$$

$$c_{i,j}^{\xi,\alpha,e} \leq x_{ii,jj}^{\xi}, \quad \forall i,j\in ACT_i, SEC_{i,j,jj}=1, \xi\in\Xi, \alpha\in\mathbb{A}, e\epsilon E \quad (26)$$

$$c_{i,j}^{\xi,\alpha,e} \leq x_{ii,j}^{\xi} + L\left(1-y_{i,ii,j}^{\xi}\right), \ \forall i,ii\in ACT_i, j\in MA_i, i\neq ii, \xi\in\Xi, \alpha\in\mathbb{A}, e\epsilon E \quad (27)$$

$$x_{ii,j}^{\xi} + \left[P_{(3)ii,j} + (2\alpha-1)\left(P_{(4)ii,j} - P_{(3)ii,j}\right)Q_{ii,j} + ST_j\right]w_{ii,j}^{\xi} \leq x_{i,j}^{\xi} + Ly_{i,ii,j}^{\xi}, \quad \forall i,ii\in ACT_i, j\in SM_i, i\neq ii, \xi\in\Xi \quad (28)$$

$$c_{ii,j}^{\xi,\alpha,e} \leq x_{i,j}^{\xi} + Ly_{i,ii,j}^{\xi} + L*(1-w_{i,j}^{\xi}), \ \forall i,ii\in ACT_i, j\in PA_j, i\neq ii, \xi\in\Xi, \alpha\in\mathbb{A}, e\epsilon E \quad (29)$$

$$xx_{i,j}^{\xi} = x_{i,j}^{\xi}, \quad \forall i\in ACT_i, j\in PA_j, \xi\in\Xi \quad (30)$$

$$xx_{i,j}^{\xi} - Lw_{i,j}^{\xi} \leq 0, \quad \forall i\in ACT_i, j\in PA_j, \xi\in\Xi \quad (31)$$

$$xx_{i,j}^{\xi} - x_{i,c}^{\xi} \leq 0, \quad \forall i\in ACT_i, j\in PA_j, \xi\in\Xi \quad (32)$$

$$x_{i,c}^{\xi} - xx_{i,j}^{\xi} + Lw_{i,j}^{\xi} \leq L, \quad \forall i\in ACT_i, j\in PA_j, \xi\in\Xi \quad (33)$$

$$\sum_{j\in PA_j} w_{i,j}^{\xi} = 1, \quad \forall i\in ACT_i, \xi\in\Xi \quad (34)$$

$$\sum_{i\in ACT_i}\sum_{j\in MA_i} w_{i,j}^{\xi} = F*I, \quad \forall\xi\in\Xi \quad (35)$$

$$e_i^{\xi,\alpha,e} \geq DD_i^{\alpha,e} - c_{i,j}^{\xi,\alpha,e}, \quad \forall i\in ACT_i, \xi\in\Xi, \alpha\in\mathbb{A}, e\epsilon E \quad (36)$$

$$t_i^{\xi,\alpha,e} \geq c_{i,j}^{\xi,\alpha,e} - DD_i^{\alpha,e}, \quad \forall i\in ACT_i, \xi\in\Xi, \alpha\in\mathbb{A}, e\epsilon E \quad (37)$$

$$t_i^{\xi,\alpha,e} \leq Ld_i^{\xi,\alpha,e}, \quad \forall i\in ACT_i, \xi\in\Xi, \alpha\in\mathbb{A}, e\epsilon E \quad (38)$$

Formula (22) and (23) define the mean and worst-case of the objective value under all machine breakdown scenarios. Constraint (24) calculates the number of breakdowns of every machine. Eq. (25) adds the robust counterparts of formula (1) with respect to the fuzzy credibility measure at level α and the left (l) or right (r) end-points of sets e. Constraints (25) – (38) adds the stochastic index of machine breakdown ξ which is counterparts of (1) – (14).

The aforementioned relations need to be adjusted using the extension principle in the scenario when the processing times and deadline dates are represented by ambiguous integers. As a result, the interval (0,1] is discretized with stepsize $SS = 1/(\mathbb{A}-1)$ to points $\{\alpha_1, \alpha_2, ..., \alpha_{\mathbb{A}}\}$, and timing calculations are made at several of the different a-levels to generate. In processing time and due date parameters, the subscripts e denote the left and right end-points utilized in the interval-arithmetic; for example, $e \in E = \{l, r\}$.

V. Multi-Objective Optimization

A. Minimize the cost of tardiness and unitary storage cost

Research on scheduling has attempted to simulate actual production environments by adding constraints from actual surroundings to the scheduling issue [37]. On the one hand, there is a growing interest in scheduling problems pertaining to minimizing tardiness and unitary storage cost [38].

$$f_1 = Minimize \sum_{i\in I}(e_i SC_i + t_i PC_i) \quad (39)$$

Earliness and tardiness can be calculated as Eq. (16), (17) and (18). Since tardy delivery could potentially have a negative impact on customer satisfaction, the business's brand and cause penalties, while early job completion may result in higher inventory storage costs [39]. Therefore, the important objective of the model is presented as (39) in order to minimize the cost of tardiness and unitary storage cost, where SC_i and PC_i are the late job penalty cost and the unitary storage cost per unit of time for job i, respectively.

B. Maximize the Overall Equipment Effectiveness (OEE)

According to Muchiri and Pintelon [40], effectiveness may be characterized as a process KPI that shows how closely the process output complies with the specifications. This is consistent with the definition found in the 378 literature, which claims that OEE evaluates the equipment's performance and quality based on the combined effects of availability, performance and quality [41, 42]. Numerous consultants and publications are available to assist managers

in using this measure since it has resulted in many beneficial impacts [43].

TABLE II. DEFINITIONS OF OEE CONCEPTS AND VARIABLES

	Minimize	Formula
Quality (Q)	Late jobs	$\frac{Delivered\ jobs\ before\ due\ time}{Total\ delivery\ orders}$
Performance (P)	Storage time	$\frac{Sum\ of\ jobs'\ completion\ time}{Sum\ of\ jobs'\ delivery\ time}$
Availability (A)	Setup/adjust	$\frac{Makespan - Total\ adjustment\ time}{Makespan}$
OEE		$A * P * Q$

Nonetheless, the definition of OEE definitions can be modified for implementation in specific circumstances [44]. For instance, compares the definitions of this rate provided by Nakajima [45] and De Groote [46] have been modified depending on the context of application. Similarly, to assess task scheduling with earliness and tardiness times, our method redefines partial rates of OEE presenting in Tables II [47]. Therefore, in this case, in order for the plant to maximize the OEE, the Eq. (40), (41) and (42) minimize the late jobs, the total storage time and the makespan respectively.

$$f_2 = Minimize \sum_{i \in I} d_i \tag{40}$$

$$f_3 = Minimize \sum_{i \in I} e_i \tag{41}$$

$$f_4 = Minimize \sum_{i \in I} \left(c_{i,J} - x_{i,1}\right) \tag{42}$$

C. Minimize the CO2 emissions

The amount of carbon dioxide (CO2) emissions is one of the most widely recognized indicators of ecological sustainability. Given the growing concern about climate change and global warming, business are under increasing pressure to reduce CO2 emissions. Managers may consider additional objectives while making a scheduling choice, in addition to minimize the CO2 emissions.

When a batch goes through processing on a machine, an amount of CO_2 emissions per unit time of processing time EC_j has produced. Moreover, in this casestudy, the company has different transportation method whether the jobs finished on time or late. Therefore, the model considers the CO_2 emissions for each late job sequenced FC and for each normal job finished NC. For a certain batching schedule PA_j, the total amount of CO_2 produced in processing all jobs and during the transportations are as follows:

$$f_5 = Minimize \sum_{i \in I} \sum_{j \in PA_j} \left(P_{ij} Q_{ij} w_{ij} EC_j + d_i FC + (1 - d_i) NC\right) \tag{43}$$

D. A hybrid RFSP ε-Constraint method

Given the requirements of the case study and the decisions that must be made to achieve multiple objectives using the model and the five objective functions described in the preceding section. Among all the objectives, the cost of tardiness and unitary storage cost are the important objective function that requires to be minimized as much as possible. Moreover, the OEE metrics and total amount of CO2 emissions is often have a specific setting target that expects the entire production environment not to exceed that standard. Therefore, the ε-constraint approach was selected to generate the results.

ε-Constraint method has been providing remedies for its well-known pitfalls since its first published by Haimes et. al. [48]. This technique will first take the highest-priority function and restricting the rest of the objectives within user-specific values ε_m as shown in (44).

$$\begin{aligned} &Minimize\ f_\mu(x) \\ &s.t.\ f_m(x) \le \varepsilon_m, m = 1, 2, \dots, M; m \ne \mu \end{aligned} \tag{44}$$

Consequently, the remaining presents in (39)-(43) can be converted to (45)-(49).

$$\begin{aligned} &Min\ M\left(e_i^{\xi,\alpha,e} SC_i + t_i^{\xi,\alpha,e} PC_i\right) \\ &+ \rho . W\left(e_i^{\xi,\alpha,e} SC_i + t_i^{\xi,\alpha,e} PC_i\right) \\ &+ \left(\frac{SS}{3}\right)\left(\frac{1}{2}\right) \sum_{\xi \in \Xi} \pi_\xi \sum_{i \in I} \sum_{\alpha \in \mathbb{A}} C_\alpha (e_i^{\xi,\alpha,L} SC_i \\ &+ t_i^{\xi,\alpha,L} PC_i + e_i^{\xi,\alpha,R} SC_i + t_i^{\xi,\alpha,R} PC_i) \end{aligned} \tag{45}$$

subject to.

$$\sum_{i \in I} d_{i,}^{\xi,\alpha,e} \le \varepsilon_{m=2}, \forall \xi \in \Xi, \alpha \in \mathbb{A}, e \in E \tag{46}$$

$$\sum_{i \in I} e_{i,}^{\xi,\alpha,e} \le \varepsilon_{m=3}, \forall \xi \in \Xi, \alpha \in \mathbb{A}, e \in E \tag{47}$$

$$\sum_{i \in I} \left(c_{i,J}^{\xi,\alpha,e} - c_{i,j=1}^{\xi,\alpha,e}\right) \le \varepsilon_{m=4}, \forall \xi \in \Xi, \alpha \in \mathbb{A}, e \in E \tag{48}$$

$$\sum_{i \in I} \sum_{j \in PA_j} \left(P_{i,j} w_{i,j}^{\xi} Q_{i,j} EC_j + d_{i,}^{\xi,\alpha,e} FC + (1 - d_{i,}^{\xi,\alpha,e}) NC\right) \le \varepsilon_{m=5},\ \forall \xi \in \Xi, \alpha \in \mathbb{A}, e \in E \tag{49}$$

Eq. (45) adds in the the area compensation integral. This variable can be used to compare the objective functions of potential schedules. Simpson coefficients [49] are employed in the integral approximation. For example, if we adopt a 7-point approximation, i.e. carry out interval arithmetic computations at $\alpha = \epsilon, 1/6, 2/6, 3/6, \dots, 1$, the set of Simpson coefficients would be $C_\propto = \{1, 4, 2, 4, 2, 4\}$.

VI. RESULT AND DISCUSSIONS

A. The Casestudy in German factory in an engine component industrial

The manufacturing operations of a German firm encompass the production of more than 150 distinct part numbers. These parts need to be scheduled efficiently to accommodate unpredictable and fluctuating demand. The production planner faces the challenging task of managing customer-requested changes to delivery dates and quantities, coordinating with the firm's supplier for materials requisition, and overseeing inventory management for finished goods, raw materials, and work in process. Additionally, the decision-making process is further complicated due to the involvement of a large number of parts and the occurrence of potential issues along the production line.

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The plant adopts a make-to-order concept, wherein the final product is manufactured only upon receiving a customer order. Specifically, the plant specializes in producing a steel-made engine component for the automotive industry. Each product comprises numerous small steel components, often numbering in the hundreds. These components undergo a series of processing steps within five distinct workstations, which are representative of common processes utilized in the steel fabrication industry. Figure 3 illustrates the arrangement of these workstations. Notably, each workstation houses multiple parallel machines with unrelated processing times. Upon completion of all necessary processing steps, the finished products are stored in a dedicated finished product storage area. Subsequently, as the due date approaches, the products are retrieved from storage and prepared for shipment to clients. This complex manufacturing environment necessitates meticulous scheduling and coordination to ensure timely delivery of products while optimizing resource utilization. The interplay between the various stages of production, the extensive number of components involved, and the intricate workflow within each workstation necessitate a comprehensive and efficient scheduling approach.

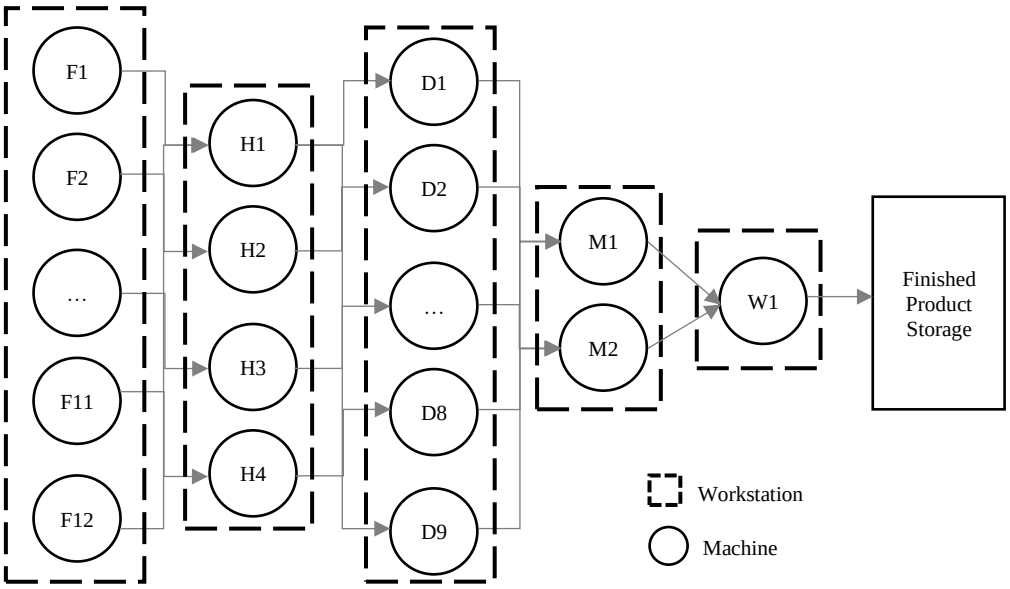


Fig. 3. The production environment of case study

The plant's specific characteristics suggest that the recommended issue aligns with the F7(Pm1,Pm5)|ddj|° category, as identified by Phadnis *et al.* [50]. This category pertains to the parallel machine flow-shop scheduling problem. In this particular scenario, the facility comprises m workstations, and all jobs are processed in the same predetermined order. The complexity is further heightened by the presence of parallel machines at two stages within the manufacturing process. Figure 3 illustrates the production environment, providing a visual representation of the setup.

Within the company's manufacturing facility, there are a total of five workstations: fine blanking, hardening, deburring, mixing, and washing. The number of machines present in each workstation is as follows: 12 machines in the fine blanking workstation, 4 machines in the hardening workstation, 9 machines in the deburring workstation, and 2 machines in the mixing workstation. The washing step, on the other hand, consists of a single-machine workstation. Adhering to the flow-shop characteristics, every job must undergo processing on one machine at each workstation in order to reach completion.

On a weekly basis, the production planner faces the task of generating a production schedule for the subsequent two weeks. This schedule typically encompasses around 20 jobs, taking into account customer demands and production capacity information. The planner must carefully consider these factors to ensure efficient utilization of resources and timely fulfillment of customer requirements.

B. Computational results

The fuzzy set-up times are given as triplets (A, B, C) that are equivalent to a TFN description. Therefore, A denotes the optimistic estimate for the lower bound of the processing time, B is the most likely value, and C is the upper bound. Tables III include the example's processing time in TFN format. Moreover, the fuzzy due date of the sample run is written as $(55,60,65)$, $(48,52,58)$ and $(65,75,80)$ for Job 1, 2 and 3 respectively.

For each problem in these models, two runs were carried out: (1) utilized a discretization of seven-point method to evaluate the integral, while (2) used a discretization of 21-point. In order to implement the MILP model, CPLEX 7.0 was used to solve them on a CPU 11th Gen Intel® Core™ i5-1135G7 @ 2.40GHz computer with 16GB RAM and WDC WDS120G2G0B-00EPW0 along with Intel® Iris® Xe Graphics. The time limit was set at 600 CPUs, and all CPLEX options were left at their default settings. For all cases, the computational test is run with the case-study data with totally 32 workstations and machines, which match with the real-life situation. The computational outcomes for the RFSP models are presented in Table IV. The number of variables, iterations, optimality gap and total processing time for the scale of 3, 7, 13, 20 and 26 jobs by implementing the 7-point and 21-point model are noted in the table.

Initially, we focus on the analysis of the seven-point models. It is evident that the computational time required to achieve optimality gap can vary significantly depending on the

problem's data. Nevertheless, the solutions obtained using the 7-point model demonstrate a remarkable proximity to optimality compared to the 21-point method. In the case of a 20-job problem, even though the time limit of 600 CPU-seconds was exceeded, the optimality gap was merely 4.67%.

This finding is noteworthy considering that the average number of jobs in a weekly schedule, as observed in the real-life case study, is approximately 20. Furthermore, in unforeseen scenarios, the number of jobs may increase up to 26. However, the optimality gap observed in the 26-job test is 6.23%, which is within an acceptable range. Thus, the computational results obtained from the seven-point model are deemed suitable for the specific case study problem.

On the other hand, when it comes to the 21-point models, the outcomes are less favorable. As expected, the CPU time required to solve these models is higher compared to the seven-point models. Only the smaller cases, namely the three-job, seven-job, thirteen-job, and twenty-job models, yielded integer solutions within the allotted time. Although these models exhibited relatively lower optimality gaps of 2.32%, 4.78%, and 6.84% for the three-job, seven-job, and thirteen-job instances, respectively, the computational time required for their completion was 520.20, 577.45, and 1033.92 seconds. However, when scaling up to the 20-job problem, the 21-point model yielded a solution with a 12.35% optimality gap, requiring 1527.72 seconds, which represents an increase of 493.8 seconds in comparison.

TABLE III. TFN DESCRIPTIONS FOR SET UP TIME OF EACH MACHINE ON EVERY WORKSTATION

Machine	Workstation 1	Workstation 2	Workstation 3	Workstation 4	Workstation 5
1	*(1.2, 2, 3)*	*(1.2, 2, 3)*	*(1.2, 2, 3)*	*(1.2, 2, 3)*	*(1, 2, 3)*
2	*(0.6, 1, 1.5)*	*(0.6, 1, 1.5)*	*(0.6, 1, 1.5)*	*(0.6, 1, 1.5)*	
3	*(1, 2, 3)*	*(1, 2, 3)*	*(1, 2, 3)*		
4	*(1.1, 2, 2.1)*	*(1.1, 2, 2.1)*	*(1.1, 2, 2.1)*		
5	*(1.1, 1.5, 2.1)*		*(1.1, 1.5, 2.1)*		
6	*(0.6, 1.5, 2)*		*(0.6, 1.5, 2)*		
7	*(0.2, 1, 1.5)*		*(0.2, 1, 1.5)*		
8	*(1, 1, 3)*		*(1, 1, 3)*		
9	*(0.5, 1, 2.1)*		*(0.5, 1, 2.1)*		
10	*(0.2, 1, 0.9)*				
11	*(0.6, 1.5, 2)*				
12	*(0.5, 1, 2.1)*				

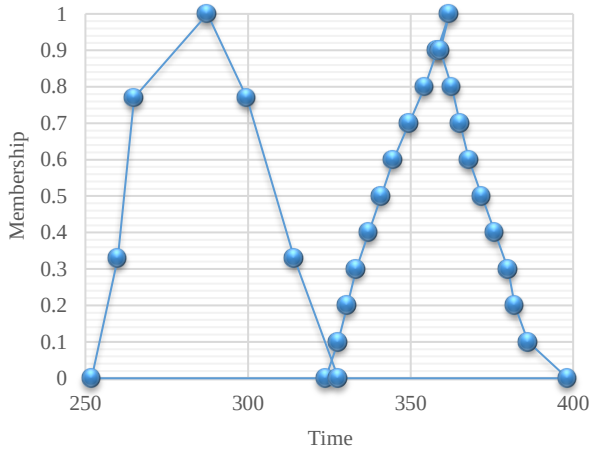


Fig. 4. Membership Function for Makespans of 7-point and 21-point discretization

The same membership function was derived using both the 21-point and 7-point models, as depicted in Figure 4 from left to right. Notably, these models were formulated with tight settings, ensuring accurate and precise results. The graph illustrates the distinct α-levels employed for the makespan value, which yield triangular results. It is evident that the 7-point model outperforms the 21-point model in terms of achieving more optimal results within the given computational time limit of 600 CPU-seconds. From a practical standpoint, the solutions obtained from the seven-point models are deemed suitable, as they exhibit minimal inaccuracies in predicting the C_α (confidence level α) of the objective value.

C. Sensitivity analysis of worst-case importunacy coefficient

Depend on the decision makers, the weighted worst-case importunacy coefficient can be different. The worst-case importunacy coefficient represents the degree of uncertainty in the scheduling problem and plays a crucial role in evaluating the robustness of the proposed approach.

TABLE IV. COMPUTATIONAL RESULTS FOR RFSP OPTIMIZATION MODELS

	3 jobs		***7 jobs***		***13 jobs***		***20 jobs***		***26 jobs***	
Model characteristics	*7-point model*	*21-point model*	*7-point model*	*21-point model*	*7-point model*	*21-point model*	*7-point model*	*21-point model*	*7-point model*	*21-point model*
Variables	190315	522577	380630	940639	510459	1405592	829530	1963638	1563742	-
Iterations (times)	492729	797427	595656	993513	794556	1293623	936952	1759684	1174954	-
Optimality Gap (%)	0.00	2.22	0.82	3.98	1.37	6.84	4.67	12.35	6.23%	-
Total processing time (secs)	210.08	520.20	260.17	577.45	294.15	1033.92	469.40	1527.72	980.15	-

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Nevertheless it might additionally be beneficial investigating a variation of every scenario in the manufacturing environment. Those making decisions will be able to see the wider context of how production and order constraints affect their scheduling process' profitability and effectiveness. Therefore, to conduct a sensitivity analysis, seven value for the worst-case importunacy coefficient have tested, starting from zero to six.

From Table V, we observe that the worst-case importunacy coefficient varies from 0 to 9 in increments of 1. The computational results demonstrate how changes in the importunacy coefficient impact the performance measures, including the makespan, energy costs, and overall equipment effectiveness (OEE). Overall, the larger of worst-case importunacy, the bigger mean-case of tardiness and unitary cost. In contrast, the worst-case is getting a better results with the increased importunacy coefficient. However, both of the value terminates their variation since $\rho = 6$.

TABLE V. COMPUTATIONAL RESULTS FOR WORST-CASE IMPORTUNACY COEFFICIENT SENSITIVITY ANALYSIS

ρ	Tardiness and unitary cost (mean-case) ($)	Tardiness and unitary cost (worst-case) ($)	Quality (%)	Performance (%)	Availability (%)	OEE (%)	CO2 emissions (tons)
0	318.75	505	72	93	95	63.61	80.21
1	437.5	494	72	93	93	62.27	84.53
2	464.7	486	72	93	90	60.26	87.64
3	477.5	489	72	93	89	59.59	90.64
4	468.75	473	72	93	91	60.93	88.53
5	448.75	463	72	93	92	61.60	87.03
6	448.75	463	72	93	92	61.60	87.03

For each increment of the importunacy coefficient, we analyze the corresponding scheduling outcomes. We observe that as the importunacy coefficient increases, all of the objectives generally tend to perform better as well. This is expected, as higher uncertainty leads to more challenging scheduling scenarios and potentially longer processing times. However, it is important to note that the increase in objective functions is not always linear, and the impact may vary depending on the specific characteristics of the scheduling problem.

VII. CONCLUSION

A multi-objective mathematical model to solve the Flow-Shop Scheduling with Parallel Machines problem (FSSPM) has proposed. Moreover, this study presents the development of robust schedule to handle the inherent uncertainty along with consideration of machine breakdowns and operational risks. Overall, the proposed optimization model has tested with the real-life case study of a German firm and shows a good results.

Analyzing 20 selected studies in the field of flow-shop scheduling under uncertainty. The review revealed various approaches used to handle uncertainty, including probabilistic and non-probabilistic methods, scenario-based techniques, and inverse uncertainty. The literature review also highlighted the different uncertain parameters considered, such as processing time and due dates, as well as the objective functions targeted, ranging from minimizing makespan to optimizing energy costs and overall equipment effectiveness. This study is the first ever FSSPM study that focuses on utilizing the triangular fuzzy approach to address both internal and external inherent uncertainties. Additionally, it takes into account scenario-based machine breakdowns and preventive maintenance, while also applying a real-life case study.

This study has suggested an MILP model that falls into the $F7(Pm_1, Pm_5)|dd_j|°$ scheduling problem. The model considers the parallel machines in flow-shop manufacturing environment. However, the optimization model is referred by Cortés *et al.* [51] associated with advanced adjustments in parameters to fit with the real-life case study objectives and the FSSPM approach. The FSSPM problem has been investigated under machine breakdowns and the inherent uncertainty of set-up time and due date parameters for the first time. This study has treated breakdowns occur as a finite set of scenarios with specific possibilities and the operational uncertainties as triangular fuzzy numbers.

A multi-objective problem has developed in this study. We have considered the cost of tardiness and unitary storage cost, the OEE measurements and the CO_2 emissions that relates to multiple departments' needs in the real-life company. This paper provided an approach for measuring success in scheduling jobs with earliness/tardiness costs and variable setup durations using the OEE metric. Moreover, we uses a hybrid RFSP ε-Constraint method to solve the MILP model.

Finally, a real-world case study in a German plant were conducted to verify problem sizes and validate the suggested approach. The results shows that the applicability and efficiency of the model is supportive and suitable with the needs of company. Moreover, the trade-off analysis of seven-point and twenty-one-point approach in a non-probabilistic fuzzy approach has implemented. The seven-point method show a greater optical for practical purposes. Furthermore, a sensitivity of worst-case importunacy coefficient have conducted to investigate the variation of objective values. A provided insights shows comprehensible relation between the mean and worst-case variables among the machine breakdown scenarios.

Future research can further explore the applicability of the RFSP model in different industrial sectors and investigate additional uncertain parameters that may impact scheduling performance. Furthermore, the development of advanced algorithms and optimization techniques can enhance the computational efficiency and scalability of the proposed model. By continuously advancing research in flow-shop scheduling under uncertainty, we can contribute to the development of more robust and efficient scheduling approaches in practical industrial systems.

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