

An Analytical Study of Suspension Bridge Collapse Based on the Characteristics of the Eigen Values

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Abstract—This research discusses an analytical approach to suspension bridge accidents focusing on eigenvalue characteristics. This research uses eigenvalue analysis to explore patterns that may contribute to bridge structural failure. The analysis steps include reducing the model to an uncoupled and linear system, as well as determining homogeneous and non-homogeneous solutions, which helps identify accident-prone patterns. The analysis results also reveal significant differences in dynamic characteristics between safe bridges and those that experience accidents. This research highlights the importance of a deep understanding of eigenvalues in designing, maintaining, and managing suspension bridges. The implication of this research is the development of more effective prevention strategies to improve suspension bridge safety in the future. By analyzing eigenvalues, this research provides deep insight into the factors that influence the stability of suspension bridges. These findings provide a strong foundation for developing recommendations for more effective design, maintenance, and management of suspension bridges. The conclusions of this research can help improve the safety and reliability of suspension bridges worldwide and contribute to civil engineering and infrastructure security.

Index Terms— Engineering, Mathematical models, Suspension bridges, Eigenvalues

I. INTRODUCTION

Bridges are fundamental components of our infrastructure, facilitating transportation and connectivity [1]. However, their collapse can lead to devastating consequences, including loss of life, property damage, and significant economic impact [2]. Therefore, understanding the factors contributing to bridge collapse is paramount [3]. This paper, titled “An Analytical Study of Bridge Collapse Based on The Characteristics of The Eigen Values,” aims to delve into the complex relationship between the

eigenvalues of a bridge’s structural matrix and its susceptibility to collapse. Eigenvalues, in the context of structural engineering, represent the dynamic response of a structure and can provide valuable insights into its stability [4]. We hypothesize that specific characteristics of these eigenvalues may serve as early warning signs of potential structural failure [5]. By studying these characteristics, we aim to develop a predictive model Huang et. al, [6] that could save lives and resources by forecasting bridge collapses Morgese et. al, [7] before they occur [8]. In the forthcoming sections, we will discuss our methodology, present our findings, and explore their implications for bridge design and maintenance [9]. We hope that our research will contribute to the ongoing efforts to enhance the safety and reliability of bridges worldwide.

A. Basic Concepts of Eigen Values

Before delving into analytical studies regarding bridge collapses, it is a must to understand the basic concept of eigenvalues. In mathematics, an eigenvalue is a scalar number associated with a non-zero vector called an eigenvector [10], [11]. This vector satisfies a specific matrix equation. In a structural context, eigenvalues are closely related to vibration analysis and structural stability [12]. In structural vibration analysis, eigenvalues determine a structure's frequency and vibration mode [13]. Each eigenvalue correlates with a particular vibration mode that indicates the deformation pattern of the structure at a specified frequency [14]. Understanding eigenvalues and vibration modes is significant in designing structures that can withstand vibrations, such as bridges, tall buildings, or other structures exposed to dynamic loads.

On the other hand, in structural stability analysis,

eigenvalues also have an essential role. Eigenvalues are related to the stability of the structure against specific external forces. A negative eigenvalue indicates the potential for structural failure due to instability [15], [16]. Therefore, identifying and evaluating eigenvalues is an essential step in assessing the safety of a structure.

B. The Relevance of Eigen Values in Bridge Structural Analysis

In the structural context of bridges, eigenvalues have significant relevance in safety analysis. As structures are exposed to various dynamic loads such as vehicle traffic, wind, and earthquakes, bridges require a comprehensive understanding of eigenvalue characteristics to ensure their stability and safety [17], [18], [19]. One of the critical applications of eigenvalues in bridge structural analysis is dynamic vibration modeling [20]. Engineers can design bridges capable of effectively resisting dynamic loads by identifying eigenvalues and associated vibration modes. For example, vibration modes related to the natural frequency of the wind can be considered in structural design to prevent resonances that could damage the structure. In addition, eigenvalues also play an essential role in analyzing the structural stability of bridges against external loads such as earthquakes. By estimating the eigenvalues associated with vibration modes susceptible to earthquakes, engineers can identify weak points in bridge structures and take appropriate preventive measures [21].

C. Implications of the Application of Eigen Values in Preventing Bridge Collapses

Analytical research on the potential for bridge collapse based on eigenvalue characteristics has significant implications for preventing structural failure [2], [22]. With a deep understanding of the eigenvalues of a bridge structure, engineers can identify weak points that are vulnerable to various types of dynamic and external loads. One of the implications of applying eigenvalues in preventing bridge collapses lies in the monitoring and maintenance process [23]. By monitoring changes in eigenvalues over time, engineers can detect signs of damage or condition in a bridge structure that may threaten its safety. Corrective or strengthening actions can be taken quickly and appropriately before potentially fatal structural failure occurs. Additionally, a solid understanding of eigenvalues significantly impacts designing safer and more durable bridge structures.

Nonlinear dynamical systems are a field of mathematical modeling that is still complicated today. One of them is a nonlinear jump system, which is often the cause of solution incompatibility [24], [25], [26]. It usually happens in suspension bridge modeling. One of the nonlinear aspects is caused by the presence of support cables, which restrain the downward movement of the bridge's middle span but have no effect in the opposite direction [27]. Because it is essential to analyze the deflection behavior formed from this torsion movement, the McKenna model is formulated by tying the beam to two hanging ropes, as in Fig. 1.

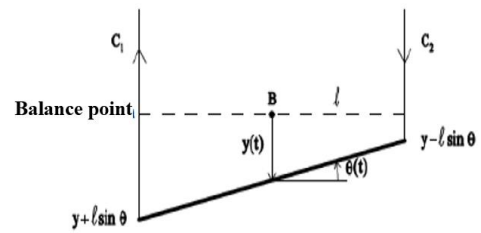


Fig. 1. Vertical torque movement [28]

Several videos on the web (UMPG Publishing) show the collapse of the Tacoma Narrows bridge, which was the most impressive failure in bridge history. However, this should have been considered because many other bridges have collapsed [29]. According to Do et. al, [22] around 400 recorded bridges failed for several reasons, and more than 70 failed after 2000. Aerodynamic instability is caused by the length of the bridge [30]. Therefore, a reliable mathematical model is necessary to describe suspension bridges' instability and future structural behavior.

Various possibilities in modeling the behavior of suspension bridges will contribute to the field of engineering [31]. It is related to safe parameters in bridge construction [27]. So, to anticipate this problem, a mathematical model linearization process is needed with asymptotic stability values and a minimum damping amount representing less material costs [32]. Pakzad and Fenves [33] stated that the identification of the attenuation estimation system in their research had yet to achieve an accuracy value. The proposal in this paper is to consider the characteristic effects of the eigenvalues, which will influence the properties of the bridge based on the damping value given to the model.

This paper examines the behavior of nonlinear equations and how hyperbolic properties influence the linearization results. If the situation is at equilibrium, then the eigenvalues and linearization results will refer to a non-zero real solution [34]. Ohene et. al, [35] revealed that the damping constant's value influences the deflection angle's stable response. The greater the damping constant provided, the more stable the response will be. However, because this research is more focused on numerical solutions and eliminates the influence of external factors, we are interested in conducting an analytical study of bridge collapse based on the eigenvalue characteristics regarding the amount of damping provided.

D. Probability of Suspension Bridge Collapse

Many causes, from mistakes in design to implementation that disregard environmental circumstances, can lead to the structural failure of bridges. As an illustration, consider the Tacoma Bridge (also known as the Tacoma Narrows Bridge), which fell in 1940 due to improperly managed aeroelastic vibrations. It results from a more rigid design that needs to account for wind effects adequately [2], [35]. Although the catastrophe taught us the value of paying attention to sound design aspects, repairs, after it happened, included adding weight to the structure to reduce vibration.

Aside from that, mistakes made during project

implementation might also result in bridge failure. As an illustration, consider the building of the Sungai Liong Bridge in Bengkalis, where the contractor should have considered whether the soil was in a state that could hold the weight of the bridge. Due to this, the bridge buckled and cracked, resulting in significant losses and raising concerns about the project management's caliber [36].

In 2011, a startling event occurred on the Kutai Kartanegara (Kukar) Bridge in Indonesia. While repair was being done, the bridge collapsed, killing people and causing enormous losses to the economy and transportation system. This tragedy emphasizes the significance of project management in general and the technical aspects of bridge structures [37], [38].

To guarantee bridges' long-term safety and success, sound design concepts, cautious project execution, and efficient risk management are crucial, as bridge structural failures teach us.

II. MATERIALS AND METHOD

This research uses a qualitative approach by presenting the phenomenon of bridge collapse behavior based on mathematical models. In theory, vertical oscillations do not directly influence bridge collapse. Various types of oscillations that are indications of bridge collapse are when destructive oscillations or torsional oscillations occur, which appear suddenly due to unpreparedness in bridge construction. In Fig. 2, you can see the difference between vertical and torsional oscillations in a suspension bridge.

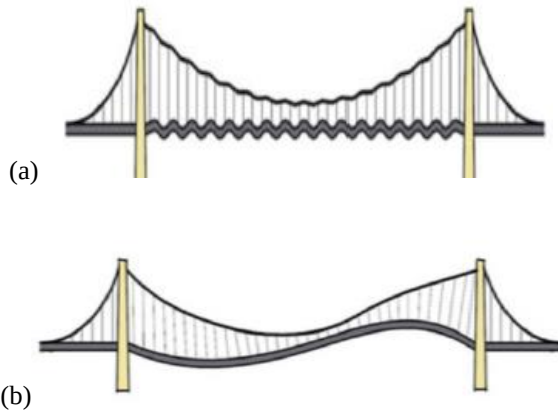


Fig. 2. Vertical (a) and torsional (b) Oscillations in suspension bridges [39]

This research outlines several steps taken to analyze the stability of a suspension bridge. The first step is to reduce the model to an uncoupled system, separating the bridge into smaller, independent parts. After that, the second step is to reduce the model to a linear system that mathematical equations can represent. Continue by determining homogeneous and non-homogeneous model solutions, where homogeneous solutions are without external influences while non-homogeneous solutions involve external influences such as wind loads or earthquakes. The next step is to interpret the solution graphically, helping to visualize the model's behavior and the effect of external influences on the bridge's stability. Finally, the research analyzes external factors such as wind loads, earthquakes,

and temperature affecting bridge stability. This approach systematically identifies factors contributing to the failure of suspension bridges and provides recommendations to improve their strength. The complete research flow is presented in Fig. 3

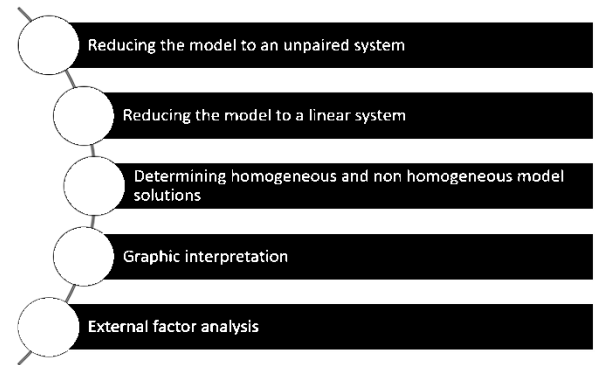


Fig. 3. Research Flow

III. RESULT AND DISCUSSION

Tajcova [27], in his research, has mentioned the paradoxical principle of cable voltage. Strengthened by Lazer and McKenna [40] with the equation derived based on vertical torque motion with the assumption that there is always a vertical cable voltage shown by equation (1),

$$\ddot{\theta} = -\frac{6k}{m} \cos \theta \sin \theta - \delta_1 \dot{\theta} + f(t) \dots\dots\dots(1)$$

By $f(t)$ is a function of time. Equation (1) is obtained based on the cable resistance from both sides due to the deflection angle. For prisoners, the left side is

$$x = \begin{cases} -k(y - l \sin \theta), & y - l \sin \theta \geq 0 \\ 0, & y - l \sin \theta \leq 0 \end{cases} \dots\dots\dots(2)$$

While the prisoners on the right are

$$x = \begin{cases} -k(y + l \sin \theta), & y + l \sin \theta \geq 0 \\ 0, & y + l \sin \theta \leq 0 \end{cases} \dots\dots\dots(3)$$

Before we discuss the following equation, recall that the $(...)^+$ is a notation to describe the behavioral state of the deflection formed by the torsional movement [40], [41]. Based on the potential energy due to vertical oscillations and influenced by the height of y causes, the potential energy is as follows,

$$E_p = \frac{1}{2} kx^2 - mgh = \frac{1}{2} k \left[\begin{aligned} & \left((y - l \sin \theta)^+ \right)^2 \\ & + \left((y + l \sin \theta)^+ \right)^2 \end{aligned} \right] - mgy \dots\dots(4)$$

While the total kinetic energy for the vertical movement of kinetic energy from the center of mass of the beam is,

$$E_{K_{\text{vertical}}} = \frac{1}{2} m \dot{y}^2 \dots\dots\dots(5)$$

With is the speed of the weight of the beam from the center of mass. To take into account the change in mass dm with length r from the center of the rod given the equation,

$$dE_{K_{dm}} = \frac{1}{2} dm (r \dot{\theta})^2 \dots\dots\dots(6)$$

Where $r \dot{\theta}$ is the linear velocity for minimal dm changes

If $dm = \frac{m}{2l} dr$ substituted together (6) with the integration limit $-l$ to l is obtained,

$$E_{K_{\text{tors}}} = \frac{m\dot{\theta}^2}{4l} \frac{1}{3} r^3 \Big|_{-l}^l = \frac{1}{6} m\dot{\theta}^2 l^2 \dots\dots\dots (7)$$

In classical mechanics, the kinetic energy of an object can be divided into two main components: translational kinetic energy, which is associated with linear translational motion, and rotational kinetic energy, which is associated with the rotational motion of the body [42]. The correlation between these two kinetic energies is that the total kinetic energy of the system, both from translational and rotational motion, can be expressed as the sum of these two components. Mathematically, this correlation can also be expressed in integral form. Integral translational kinetic energy of the system. This correlation reflects that the total kinetic energy of a system is the sum of its translational kinetic energy and its rotational kinetic energy, both in the case of a single system and more complex systems consisting of several moving objects [43]. This fundamental principle of classical mechanics is essential in analyzing and understanding the motion of physical systems. $\dot{\theta}$ is angular speed. From equation (5) and equation (7), the mechanical energy of the system is obtained,

$$E_m = \frac{1}{2} m\dot{y}^2 + \frac{1}{6} m\dot{\theta}^2 l^2 \dots\dots\dots (8)$$

If presented in the form of Lagrangian, it is obtained

$$L = \frac{1}{2} m\dot{y}^2 + \frac{1}{6} m\dot{\theta}^2 l^2 - \left(\frac{1}{2} k \left[\left((y-l\sin\theta)^+ \right)^2 + \left((y+l\sin\theta)^+ \right)^2 \right] - mgy \right) \dots\dots\dots (9)$$

by following the Euler-Lagrange equation for $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0$, then $\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{3} m\dot{\theta} l^2$, $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = \frac{1}{3} m\ddot{\theta} l^2$

and $\frac{\partial L}{\partial \theta} = kl \cos \theta \left[(y-l\sin\theta)^+ - (y+l\sin\theta)^+ \right]$. Thus

$$\frac{1}{3} m\ddot{\theta} l^2 - \left[kl \cos \theta \left((y-l\sin\theta)^+ - (y+l\sin\theta)^+ \right) \right] = 0$$

Then obtained,

$$\ddot{\theta} = \frac{3k}{ml} \cos \theta \left((y-l\sin\theta)^+ - (y+l\sin\theta)^+ \right) \dots\dots\dots (10)$$

In the context of the total energy of the system, the role of damping factors becomes urgent because it can influence how translational and rotational kinetic energy is treated in the system [24]. More excellent damping will cause the system energy to decrease more quickly, while less damping will allow energy to be maintained longer. Therefore, paying attention to the damping factor is essential to understand how the total energy of the system changes over time. If the damping factor becomes

$$\ddot{\theta} = \frac{3k}{ml} \cos \theta \left((y-l\sin\theta)^+ - (y+l\sin\theta)^+ \right) - \delta_1 \dot{\theta} + f(t) \dots\dots\dots (11)$$

$$\delta_1 \dot{\theta} + f(t)$$

Which δ_1 is a partial derivative function Whereas for

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}} \right) - \frac{\partial L}{\partial y} = 0 \text{ . was obtained}$$

$$\ddot{y} = \frac{k}{m} \left((y-l\sin\theta)^+ + (y+l\sin\theta)^+ \right) - \delta_1 \dot{\theta} + g \dots\dots\dots (12)$$

Because there is always a vertical cable voltage causes,

$$\ddot{\theta} = -\frac{6k}{m} \cos \theta \sin \theta - \delta_1 \dot{\theta} + f(t) \dots\dots\dots (13)$$

and

$$\ddot{y} = -\frac{2k}{m} y - \delta_2 \dot{y} + g \dots\dots\dots (14)$$

Equation (13) and equation (14) are influenced by conditions $y \pm l \sin \theta \geq 0$ so that $(y \pm l \sin \theta)^+ = y \pm l \sin \theta$.

with $f(t) = \beta \sin(\mu t)$ and $\cos \theta \sin \theta = \sin 2\theta$ then the angular equation that describes the torque motion is

$$\ddot{\theta} = -\frac{3k}{m} \sin(2\theta) - \delta_1 \dot{\theta} + \beta \sin(\mu t) \dots\dots\dots (15)$$

if you make a derivative θ_1 and θ_2 by definition $\theta_1 = \theta$ and $\theta_2 = \dot{\theta}$ concerning t it will be obtained $\dot{\theta}_1 = \dot{\theta}$ and $\dot{\theta}_2 = \ddot{\theta}$ so equation (15) becomes,

$$\begin{cases} \dot{\theta}_1 = \theta_2 \\ \dot{\theta}_2 = -\frac{3k}{m} \sin(2\theta) - \delta_1 \theta_2 + \beta \sin(\mu t) \end{cases} \dots\dots\dots (16)$$

To expand Taylor's equation (15), the balance point is chosen $(\theta_1^*, \theta_2^*) = (n, \pi, 0)$ with $n = 0, 1, 2, 3, \dots$ that is

$(\theta_1^*, \theta_2^*) = (0, 0)$ so the equation becomes,

$$\begin{cases} \dot{\theta}_1 = \theta_2 \\ \dot{\theta}_2 = -\frac{6k}{m} \theta_1 - \delta_1 \theta_2 + \beta \sin(\mu t) \end{cases} \dots\dots\dots (17)$$

To determine the solution of equation (17), the eigenvalue characteristic is determined first

$$\lambda_{1,2} = \frac{-\delta \pm \sqrt{\delta^2 - \frac{24k}{m}}}{2} \dots\dots\dots (18)$$

If the real eigenvalues are different $\delta^2 - \frac{24k}{m} > 0$

$$\begin{aligned} \theta_1(t) &= C_1 e^{\frac{-\delta+P}{2}t} + C_2 e^{\frac{-\delta-P}{2}t} + \\ &4\beta \left(\frac{(\delta^2 - P^2 - 4\mu^2) \sin(\mu t) - 4\mu\delta \cos(\mu t)}{(4\mu^2 + (\delta - P)^2)(4\mu^2 + (\delta + P)^2)} \right) \end{aligned} \dots\dots\dots (19)$$

$$\begin{aligned} \theta_2(t) &= C_1 \frac{P - \delta}{2} e^{\frac{P - \delta}{2}t} + C_2 \frac{-\delta - P}{2e^{\frac{P + \delta}{2}t}} + \\ &4\beta \left(\frac{(\delta^2 - P^2 - 4\mu^2) \mu \cos(\mu t) + 4\delta\mu^2 \sin(\mu t)}{(4\mu^2 + (\delta - P)^2)(4\mu^2 + (\delta + P)^2)} \right) \end{aligned}$$

$$\text{with } P = \sqrt{\delta^2 - \frac{24k}{m}}$$

$$C_1 = \frac{(P + \delta)\theta_1(0) + 2\theta_2(0)}{2P} -$$

$$\frac{4\beta\mu\left(\frac{24K}{m} - 4\mu^2\right) + 8\beta\mu\delta(P - \delta)}{P(4\mu^2 + (\delta - P)^2)(4\mu^2 + (P + \delta)^2)}$$

and

$$C_2 = \frac{2\theta_2(0) - (P - \delta)\theta_1(0)}{2P} -$$

$$\frac{4\beta\mu\left(\frac{24K}{m} - 4\mu^2\right) + 8\beta\mu\delta(P - \delta)}{P(4\mu^2 + (\delta - P)^2)(4\mu^2 + (P + \delta)^2)}$$

with graphic images presented in Fig. 4 and Fig. 5.

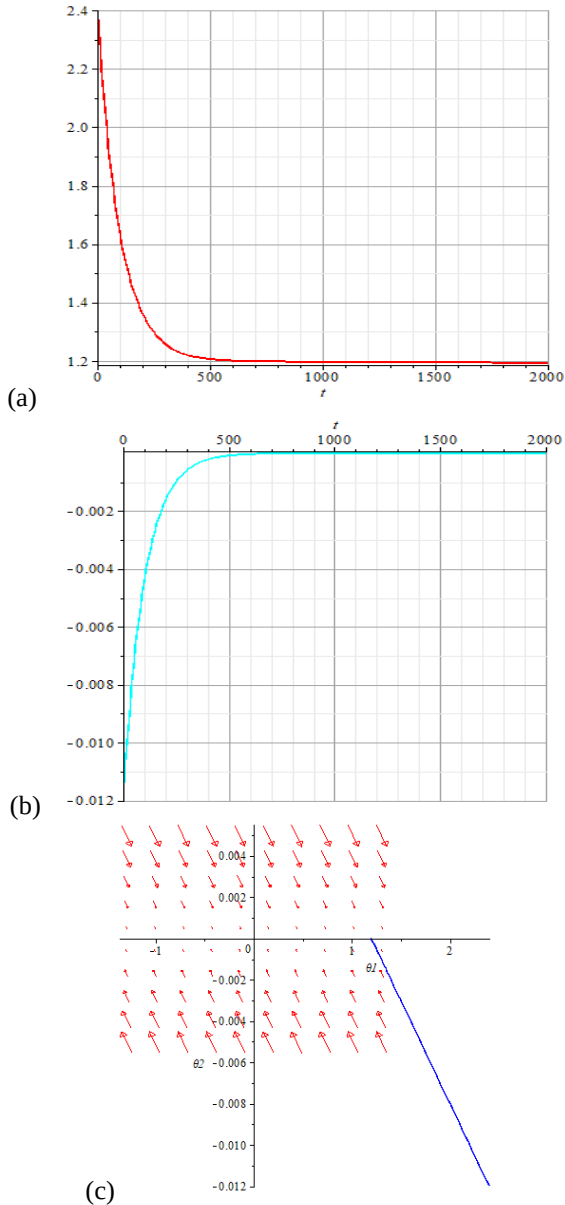


Fig. 4. The graph of a homogeneous solution of real eigenvalues is different from $k=3.75$, $m=757300000$ dengan $\delta=0.01$, (a) C_1 , (b) C_2 and (c)

$$sys = \left| \theta_2, -\frac{6k}{m}\theta_1 - \delta\theta_2 \right|$$

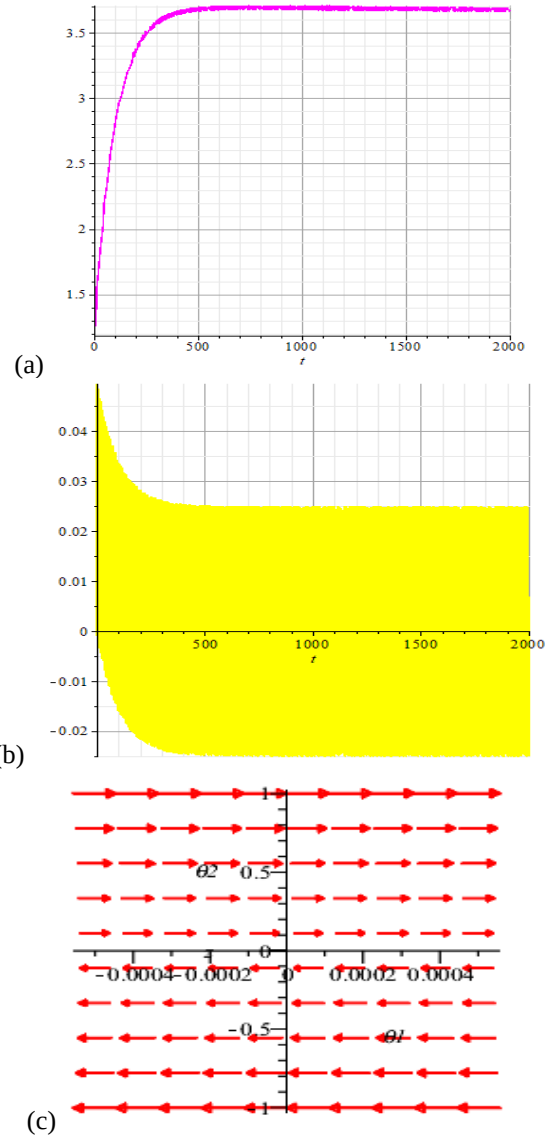


Fig. 5. The non-homogeneous solution graph of real eigenvalues is different from $k=3.75$, $m=757300000$, $\delta=0.01$, $\beta=0.04$, $\mu=1.6$, (a) C_1 , (b) C_2 and (c)

$$sys = \left| \theta_2, -\frac{6k}{m}\theta_1 - \delta\theta_2 + \beta\sin(\mu z), 1 \right|$$

If eigen is a real twin $\delta^2 - \frac{24k}{m} = 0$

$$\begin{aligned}
 \theta_1(t) &= e^{-\frac{\delta}{2}t} (C_1 + C_2) + \\
 &\quad - \frac{\beta t}{\mu^2 + \frac{\delta^2}{4}} \left(\frac{\delta}{2} \sin(\mu t) - \mu \cos(\mu t) \right) \\
 &\quad \beta \left(\frac{(4\mu t(\delta^2 + 4\mu^2) - 16\mu\delta) \cos(\mu t)}{(\delta^2 + 4\mu^2)^2} - \right. \\
 &\quad \left. \frac{(8\mu^2(2 + \delta t) + 2\delta^3(t-2)) \sin(\mu t)}{(\delta^2 + 4\mu^2)^2} \right) \\
 \theta_2(t) &= e^{-\frac{\delta}{2}t} \left(-\frac{\delta}{2} C_1 + \frac{2 - \delta t}{2} C_2 \right) + \\
 &\quad \beta \frac{2 - \beta t}{2 \left(\mu^2 + \frac{\delta^2}{4} \right)} \left(\frac{\delta}{2} \sin(\mu t) - \mu \cos(\mu t) \right) - \\
 &\quad \frac{\delta \beta}{2} \left(\frac{(4\mu t(\delta^2 + 4\mu^2) - 16\mu\delta) \cos(\mu t)}{(\delta^2 + 4\mu^2)^2} - \right. \\
 &\quad \left. \frac{(8\mu^2(2 + \delta t) + 2\delta^3(t-2)) \sin(\mu t)}{(\delta^2 + 4\mu^2)^2} \right) \dots\dots\dots(20)
 \end{aligned}$$

with

$$\begin{aligned}
 C_1 &= \theta_1(0) + \left(\frac{16\mu\delta}{(\delta^2 + 4\mu^2)^2} \right) \beta \\
 C_2 &= \theta_2(0) + \frac{\delta}{2} \theta_1(0) + \\
 &\quad \beta \left(\frac{4\mu}{\delta^2 + 4\mu^2} \frac{8\mu\delta^2}{(\delta^2 + 4\mu^2)^2} \right)
 \end{aligned}$$

with graphic images presented in Fig. 6 and Fig. 7.

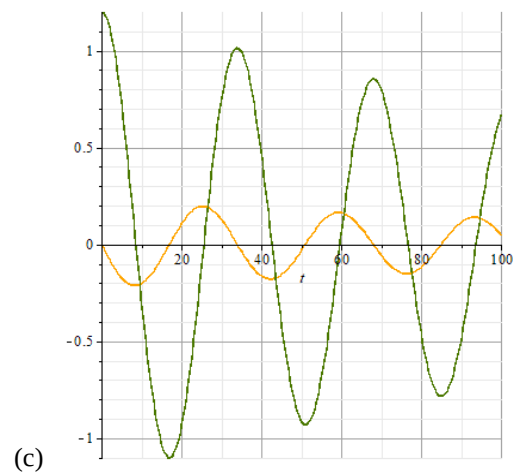
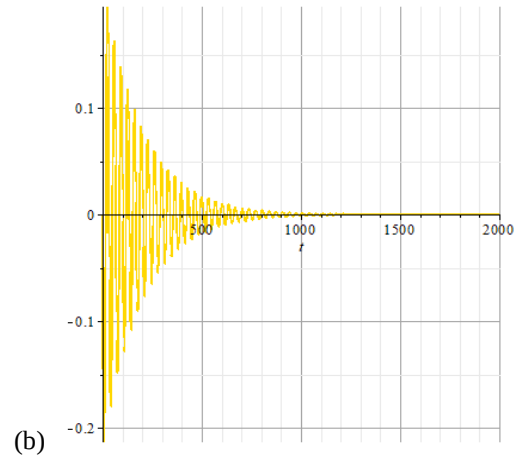
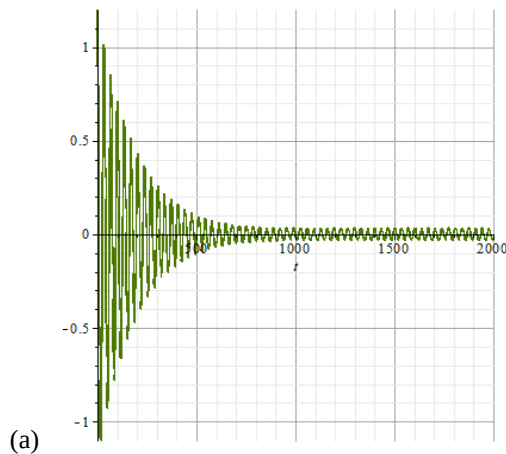
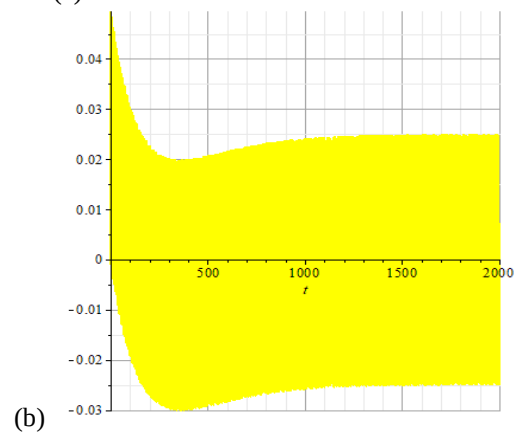
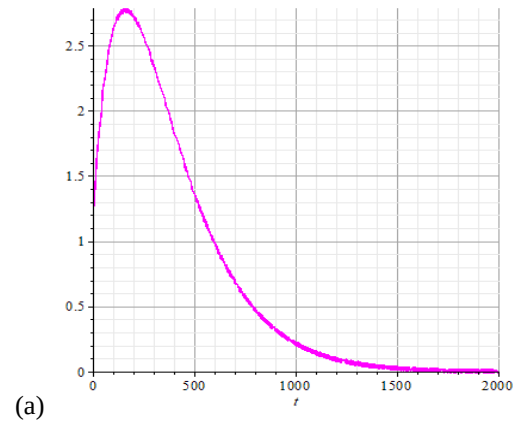
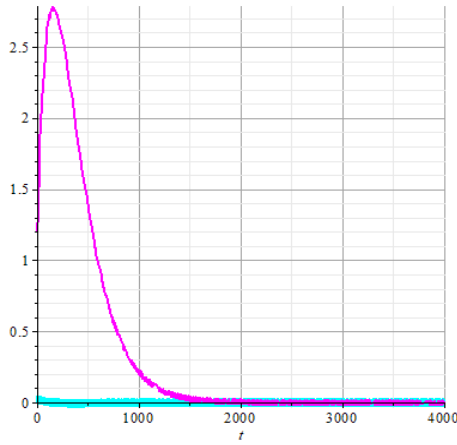
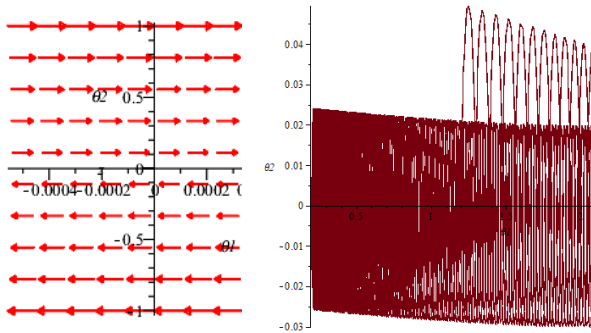


Fig. 6. Graph of a homogeneous solution of repeated (twin) eigenvalues with $k=3.75$, $m=757300000$, $\delta=0.01$, (a) C_1 , (b) C_2 and (c) combined C_1 and C_2





(c)



(d)

Fig. 7. Graph of non-homogeneous solutions of repeated eigenvalues (twins) with $k=3.75$, $m=757300000$, $\delta=0.01$, $\beta=0.04$, $\mu=1.6$, (a) C_1 , (b), C_2 and (c) combined C_1 and C_2 , (d) $\text{sys} = \left| \theta_2, -\frac{6k}{m}\theta_1 - \delta\theta_2 + \beta\sin(\mu z), 1 \right|$

If the eigen complex is real $\delta^2 - \frac{24k}{m} < 0$

$$\theta_1(t) = e^{\frac{\delta}{2}t} \left(C_1 \cos\left(\frac{Q}{2}t\right) + C_2 \cos\left(\frac{Q}{2}t\right) \right) + \frac{\beta \cos\left(\frac{Q}{2}t\right)}{Q} \left(\frac{\frac{\delta}{2} \cos\left(\left(\frac{Q}{2} + \mu\right)t\right) + \left(\frac{Q}{2} + \mu\right) \sin\left(\left(\frac{Q}{2} + \mu\right)t\right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} + \mu\right)^2} - \frac{\frac{\delta}{2} \cos\left(\left(\frac{Q}{2} - \mu\right)t\right) + \left(\frac{Q}{2} - \mu\right) \sin\left(\left(\frac{Q}{2} - \mu\right)t\right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2} \right) + \frac{\beta \sin\left(\frac{Q}{2}t\right)}{Q} \left(\frac{\frac{\delta}{2} \sin\left(\left(\frac{Q}{2} + \mu\right)t\right) - \left(\frac{Q}{2} + \mu\right) \cos\left(\left(\frac{Q}{2} + \mu\right)t\right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} + \mu\right)^2} - \frac{\frac{\delta}{2} \sin\left(\left(\frac{Q}{2} - \mu\right)t\right) - \left(\frac{Q}{2} - \mu\right) \cos\left(\left(\frac{Q}{2} - \mu\right)t\right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2} \right)$$

$$\theta_2(t) = e^{\frac{\delta}{2}t} \left(C_1 \left(\frac{-\delta}{2} \cos\left(\frac{Q}{2}t\right) - \frac{Q}{2} \sin\left(\frac{Q}{2}t\right) \right) + C_2 \left(\frac{-\delta}{2} \sin\left(\frac{Q}{2}t\right) + \frac{Q}{2} \cos\left(\frac{Q}{2}t\right) \right) \right) + \frac{\beta \left(\frac{-\delta}{2} \cos\left(\frac{Q}{2}t\right) - \frac{Q}{2} \sin\left(\frac{Q}{2}t\right) \right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2} + \frac{\frac{\delta}{2} \cos\left(\frac{Q+2\mu}{2}t\right) + \frac{Q+2\mu}{2} \sin\left(\frac{Q+2\mu}{2}t\right) - \frac{\delta}{2} \cos\left(\frac{Q-2\mu}{2}t\right) + \frac{Q-2\mu}{2} \sin\left(\frac{Q-2\mu}{2}t\right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2} + \frac{\beta \left(\frac{-\delta}{2} \sin\left(\frac{Q}{2}t\right) + \frac{Q}{2} \cos\left(\frac{Q}{2}t\right) \right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2} + \frac{\frac{\delta}{2} \sin\left(\frac{Q+2\mu}{2}t\right) - \frac{Q+2\mu}{2} \cos\left(\frac{Q+2\mu}{2}t\right) - \frac{\delta}{2} \sin\left(\frac{Q-2\mu}{2}t\right) - \left(\frac{Q}{2} - \mu\right) \cos\left(\left(\frac{Q}{2} - \mu\right)t\right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2}$$

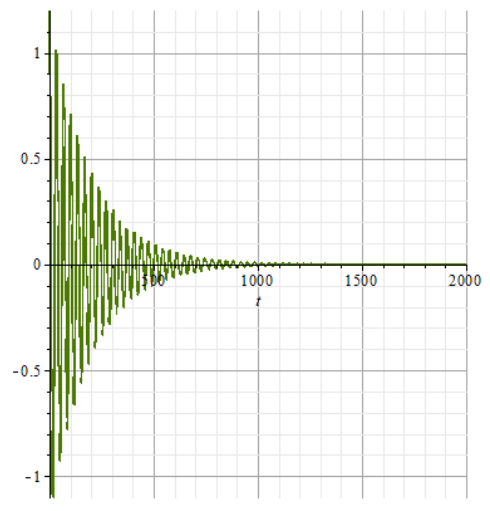
$$\theta_2(t) = e^{\frac{\delta}{2}t} \left(C_1 \left(\frac{-\delta}{2} \cos\left(\frac{Q}{2}t\right) - \frac{Q}{2} \sin\left(\frac{Q}{2}t\right) \right) + C_2 \left(\frac{-\delta}{2} \sin\left(\frac{Q}{2}t\right) + \frac{Q}{2} \cos\left(\frac{Q}{2}t\right) \right) \right) + \frac{\beta \left(\frac{-\delta}{2} \cos\left(\frac{Q}{2}t\right) - \frac{Q}{2} \sin\left(\frac{Q}{2}t\right) \right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2} + \frac{\frac{\delta}{2} \cos\left(\frac{Q+2\mu}{2}t\right) + \frac{Q+2\mu}{2} \sin\left(\frac{Q+2\mu}{2}t\right) - \frac{\delta}{2} \cos\left(\frac{Q-2\mu}{2}t\right) + \frac{Q-2\mu}{2} \sin\left(\frac{Q-2\mu}{2}t\right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2} + \frac{\beta \left(\frac{-\delta}{2} \sin\left(\frac{Q}{2}t\right) + \frac{Q}{2} \cos\left(\frac{Q}{2}t\right) \right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2} + \frac{\frac{\delta}{2} \sin\left(\frac{Q+2\mu}{2}t\right) - \frac{Q+2\mu}{2} \cos\left(\frac{Q+2\mu}{2}t\right) - \frac{\delta}{2} \sin\left(\frac{Q-2\mu}{2}t\right) - \left(\frac{Q}{2} - \mu\right) \cos\left(\left(\frac{Q}{2} - \mu\right)t\right)}{\left(\frac{\delta}{2}\right)^2 + \left(\frac{Q}{2} - \mu\right)^2}$$

with $Q = \sqrt{\frac{24k}{m} - \delta^2}$

$$C_1 = \theta_1(0) - \frac{\beta}{Q} \left(\frac{2\delta}{\delta^2 + (Q+2\mu)^2} - \frac{2\delta}{\delta^2 + (Q+2\mu)^2} \right)$$

$$C_2 = \frac{2\theta_2(0) + \delta\theta_1(0)}{Q} - \frac{\beta}{Q^2} \left(\frac{2Q^2 + 4\mu Q}{\delta^2 + (Q+2\mu)^2} - \frac{2Q^2 - 4\mu Q}{\delta^2 + (Q-2\mu)^2} \right)$$

From this assumption, the cause of the collapse of the Tacoma Narrows bridge is due to the provision of forced damping [35]. with graphic images presented in Fig. 8 and Fig. 9.



(a)

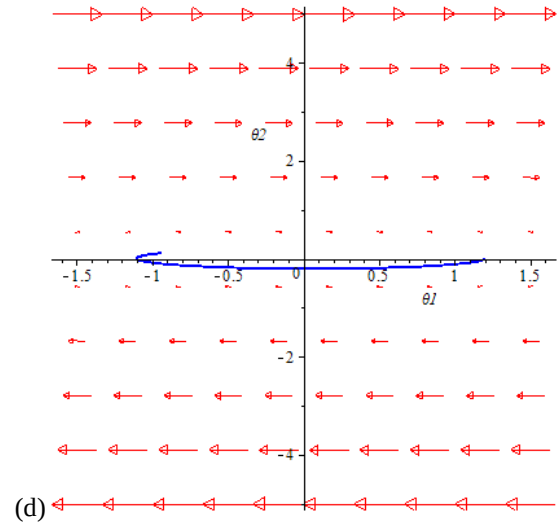
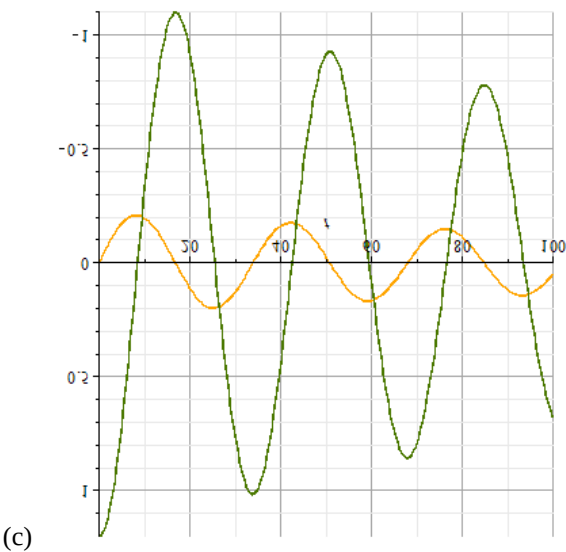
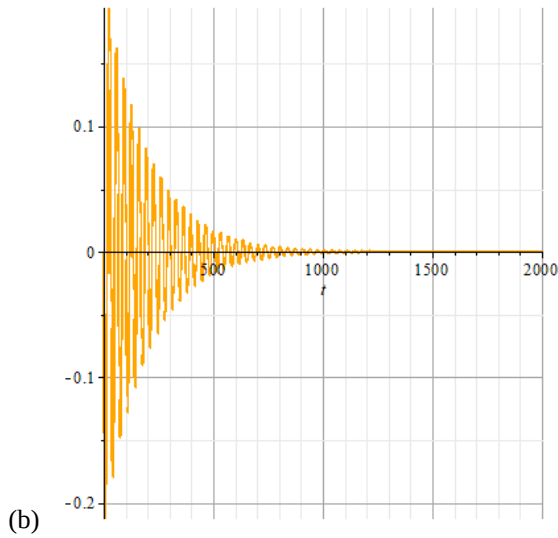


Fig. 8. Graph of a homogeneous solution of complex eigenvalues with $k=3.75$, $m=757300000$, $\delta=0.01$, (a) C_1 , (b), C_2 and (c) combined C_1 and C_2 , (d)

$$sys = \left[\theta_2, -\frac{6k}{m}\theta_1 - \delta\theta_2 \right]$$

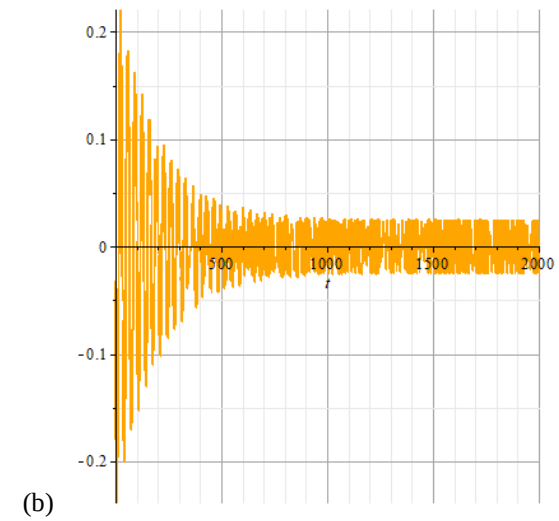
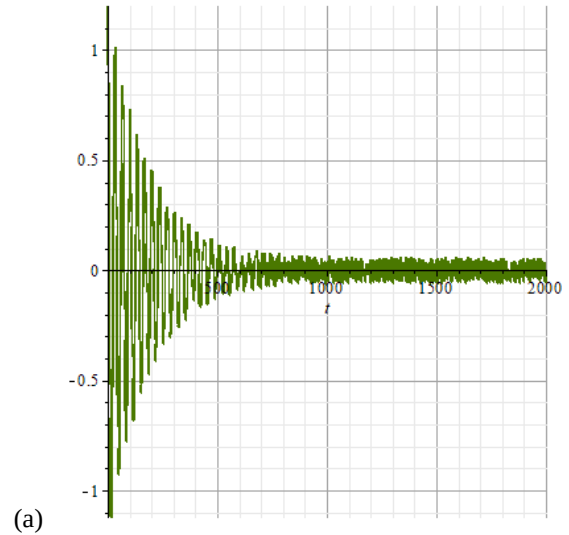


Fig. 9. Graph of non-homogeneous solutions of complex eigenvalues with $k=3.75$, $m=757300000$, $\delta=0.01$, $\beta=0.04$, $\mu=1.6$, (a) C_1 , (b), C_2

The external factor $f(t)$ has a different impact in the case of distinct real eigenvalues and twin reals than in the case of complex eigenvalues. In the phase portrait of the deflection angle model with $f(t)$, the real eigenvalues are different, and the accurate twin trajectories $\theta_1(t)$ and $\theta_2(t)$ do not approach the equilibrium point. On the other hand, in the case of complex eigenvalues, the trajectories $\theta_1(t)$ and $\theta_2(t)$ are still close to zero, even though they are chaotic. However, in the case of complex eigenvalues with $f(t)$, the graph shows a random nature, so its stability is difficult to guarantee. These findings illustrate the complexity of the interaction between external factors and eigenvalue characteristics in determining the strength of suspension bridges. This understanding is essential in designing, maintaining, and managing bridge infrastructure to minimize the risk of accidents. In addition, this research highlights the need for a holistic approach to analyzing structural dynamics, integrating external factors as an integral part of stability evaluation. In this way, prevention and repair efforts can be targeted more effectively, improving the safety of suspension bridges in the future.

Aside from that, the load on the bridge significantly impacts the structure's performance and confidentiality if it has to do with field facts. Wind, earthquakes, pedestrian traffic, cars, and other events may cause these loads. These loads must be carefully considered during the design process to guarantee that the bridge can safely handle the load for its service life [44]. The bridge may undergo overload, structural failure, or collapse if the load exceeds the planned capacity. As a result, thorough load analysis is crucial to bridge design and planning.

Patterns in a suspension bridge's eigenvalues offer a crucial foundation for designing, maintaining, and operating it. Identifying fundamental vibration modes is a primary use of eigenvalues that enables engineers to predict possible resonances with loads such as wind or other loads that may jeopardize the integrity of a structure. This identification allows structural designers to lower the failure risk by implementing particular measures. The knowledge from eigenvalues can also guide the choice of suitable materials and building methods; materials with mechanical characteristics corresponding to the recognized critical vibration modes can be chosen.

Vibration patterns can also serve as a basis for efficient maintenance. Maintenance managers can regularly monitor bridge vibration frequencies and patterns to spot potential structural changes over time [45], [46]. Traffic management can also be tailored to essential vibration patterns to lower the chance of damage or failure from excessive vibration. Implementing eigenvalue-based early warning and monitoring systems can be a helpful first step toward preserving bridge safety. This technology can provide early warning for possible issues with the bridge structure by placing sensors that can detect vibrations that correspond to the defined critical modes.

Enhancing the dependability, safety, and general performance of suspension bridges requires incorporating data from eigenvalues into the design, upkeep, and management procedures. Suspension bridges can be planned and operated more efficiently, ensuring user safety and prolonging the structure's life by identifying important vibration patterns and taking appropriate action [47].

V. CONCLUSION

From the results of this research, eigenvalue characteristic analysis is a valuable approach to understanding and analyzing the stability of suspension bridges. These findings have important implications for the development of suspension bridge accident prevention strategies in the future. Steps such as reducing the model to an uncoupled system, reducing the model to a linear system, and determining homogeneous and non-homogeneous solutions have helped identify patterns susceptible to structural failure. Graphical analysis and research of external factors also contribute to our understanding of the stability of suspension bridges. Thus, this research provides a strong foundation for developing recommendations for more effective design, maintenance, and management of suspension bridges. It is hoped that preventive measures based on these findings will improve the safety and reliability of suspension bridges worldwide. Analysis of eigenvalues on suspension bridges that experience accidents shows a consistent pattern in certain eigenvalues. These patterns may include specific natural

frequencies or vibration mode patterns susceptible to certain external loads.

In some cases, these patterns can be potential indicators for early detection of failure risks in suspension bridges. In addition, comparing the eigenvalues between accident-affected suspension bridges and safe bridges has revealed significant differences in their dynamic characteristics. It shows the importance of a deep understanding of eigenvalues in designing, maintaining, and managing suspension bridges to avoid fatal structural failures.

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