Reliability-Based Assessment of Shallow Foundation Performance Considering CPT-Derived Spatial Variability

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***Abstract*—The spatial variability of soil properties revealed by cone penetration test (CPT) data introduces significant uncertainty in foundation performance predictions. This study presents a reliability-based framework that translates CPT-derived spatial variability directly into the probability of failure of shallow foundations without finite element analysis. Statistical characterization of 16 CPT soundings at a tropical volcanic soil site yields coefficients of variation of cone resistance from 0.49 to 1.09 and vertical scales of fluctuation from 0.37 to 1.48 m. Two constitutive assumptions are compared: granular soil analyzed with the Vesic bearing capacity and Schmertmann settlement models, and cohesive soil analyzed with undrained bearing capacity and elastic settlement. Monte Carlo simulation with Latin Hypercube Sampling and 100,000 simulations shows that the serviceability limit state governs reliability, whereas the probability of bearing capacity failure remains zero. The assumed soil type, however, changes settlement reliability by a factor of twenty to forty: the failure probability ranges from 0.57 to 0.88 under the sand assumption, with negative reliability indices, and from 0.016 to 0.144 under the clay assumption, where the reliability index remains below the target range. Young's modulus dominates the uncertainty in both cases, whereas the strength parameters exert negligible influence. CPT-based soil behavior type classification is therefore a prerequisite for meaningful reliability assessment, and factor-of-safety design may be unconservative when settlement governs.**

Keywords—cone penetration test, foundation reliability, Monte Carlo simulation, reliability index, settlement, spatial variability.

I. INTRODUCTION

The design of shallow foundations requires engineers to manage substantial uncertainty arising from the spatial variability of natural soil deposits and the indirect nature of field measurements [1], [2]. The traditional approach, which applies a global factor of safety of 3.0 to the ultimate bearing capacity, conflates multiple uncertainty sources into a single number and offers no insight into which limit state governs or what level of reliability is actually achieved [3]. Although reliability-based design has been codified in structural engineering through partial factor approaches [4], [5] and adopted in geotechnical design [6], the calibration of partial factors relies on generic variability statistics from global databases [7] rather than on site-specific measurements, a limitation that has motivated recent work on measurement-specific site variability analysis [8] and on the rational determination of the number of soundings needed to capture it [9]. A site whose coefficient of variation (COV) significantly exceeds the calibrated range may achieve a reliability far below the intended target, representing a fundamental limitation of current practice that recent reliability-based design formulations attempt to overcome by operating directly on site-specific statistics [10].

The cone penetration test (CPT) provides continuous, high-resolution profiles of penetration resistance that are ideally suited to quantifying site-specific spatial variability [11]; where soundings are sparse, spatial interpolation and synthetic profile generation extend the information available for that purpose [12]. Extensive research has characterized CPT variability through the COV and the vertical scale of fluctuation (θ_v), which measures the correlation distance of soil properties [13], and COV exceeding 0.5 is classified as high variability [7]. Estimation methods for θ_v have been refined through the method of moments [14], Bayesian inference [15], kriging-based conditioning [16], machine learning [17], and probabilistic site classification that treats the sounding as a random field [18]. However, these studies typically conclude with statistical parameters without translating them into foundation reliability metrics or design decisions. Spatial averaging over the foundation influence zone attenuates the variance that governs performance, yet that attenuation is only rarely carried through to design [19].

On the foundation reliability side, the random finite element method has enabled probabilistic analysis of bearing capacity and settlement on spatially variable soils [3], [20], [21], demonstrating that spatial variability reduces mean bearing capacity and increases the probability of failure. Later analyses have quantified the influence of asymmetrical failure mechanisms [22] and of random-field discretization error [23] on the resulting capacity statistics, and three-dimensional treatments show that geological uncertainty must be modelled alongside spatial variability [24]. Yet this method requires finite element analysis, specialized software, and substantial computation, limiting its accessibility for routine design and constraining parametric sensitivity studies. In parallel, established correlations link CPT measurements to soil design parameters, including the effective friction angle in sands [25], Young's modulus [26], and undrained shear strength in clays [11]. Each correlation carries transformation uncertainty [7], [27], an uncertainty that continues to be re-assessed against independent databases for the effective friction angle [28], the undrained shear strength [29], and the soil behaviour type index [30], yet is rarely propagated through to reliability metrics.

Efforts to bridge site characterization and reliability analysis include the value-of-information framework [31],

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[32] and geostatistical site characterization [33]. However, these operate at the level of design parameter statistics rather than directly on CPT measurements, and they stop short of evaluating foundation performance. Uncertainty-aware predictive models of foundation capacity have been developed on the same principle [34], but their calibration is generic rather than site-specific. The critical remaining gap, namely an end-to-end framework linking raw CPT variability to foundation probability of failure through a computationally accessible analytical approach, remains unaddressed. Studies that optimize borehole placement under spatially variable soil [35] likewise depend on a link between investigation effort and foundation performance, which is precisely what is missing.

This study develops and demonstrates a Monte Carlo simulation framework that directly translates CPT-derived spatial variability into the probability of failure of shallow foundations, and evaluates how the choice of soil constitutive model (drained sand versus undrained clay) affects reliability conclusions. The framework is applied to 16 CPTs at a tropical volcanic soil site with high inherent variability (COV of cone resistance exceeding 0.5), conditions that challenge the assumptions underlying code-based partial factors. The contributions of this study can be summarized in three key aspects. First, it provides the first complete analytical CPT-to-reliability pipeline requiring only standard computing resources. Second, it quantifies for the first time the 20 to 40 times difference in settlement failure probability that arises solely from the sand versus clay constitutive assumption, establishing CPT-based soil classification as a prerequisite for meaningful reliability assessment. Third, it demonstrates that Young's modulus, rather than strength parameters, is the dominant uncertainty source for settlement reliability, with direct implications for site investigation prioritization and design code calibration.

II. METHODS

A. Site Description and CPT Data

The study site is located in a tropical volcanic region at Desa Taro, Ubud, Gianyar, Bali. A site investigation program comprising 16 cone penetration tests (designated S1 through S16) was conducted at 0.2 m measurement intervals. The CPTs reached depths from 3.0 m (S10) to 14.8 m (S13). All analyses use the cone resistance (q_c) values reported in kg/cm^2 as the primary measurement. The site is characterized by high inherent variability, with 15 of the 16 soundings having a COV exceeding 0.5 according to the classification of Phoon and Kulhawy [7].

Tropical volcanic soils are formed by the weathering of volcanic parent materials under tropical climatic conditions, producing heterogeneous deposits that commonly alternate between granular and cohesive layers within short vertical distances. The penetration resistance profiles at this site display the irregular, multi-layered character typical of such deposits, with abrupt transitions between soft and dense zones. This geological setting is fundamentally different from the uniform alluvial or glacial deposits that dominate the calibration databases of many CPT correlations, which is precisely why site-specific variability characterization is essential before applying reliability methods developed for other geological contexts. The close spacing of the soundings relative to the site extent allows a preliminary assessment of both vertical and lateral variability, although the layout does not permit a formal two-dimensional geostatistical analysis.

B. Statistical Characterization of CPT Data

Each CPT profile was processed through a standardized pipeline of four sequential operations. First, spike removal was performed using an interquartile range filter applied to the first-difference of q_c, on the rationale that measurement spikes manifest as sudden jumps between adjacent readings. Second, a linear trend was fitted to the q_c-depth profile and subtracted to obtain stationary residuals for spatial correlation analysis. Third, the vertical scale of fluctuation (θ_v) was estimated by the method of moments [13], integrating the sample autocorrelation function to its first crossing of e^{-1}. Fourth, distribution fitting was performed through Shapiro-Wilk tests on both the raw q_c values and their natural logarithms, with the distribution yielding the higher p-value selected as the best fit.

C. CPT-to-Soil-Parameter Transformation

Two soil models were implemented to evaluate the sensitivity of reliability conclusions to the assumed constitutive framework. The first model (V1) assumes granular soil behavior. The effective friction angle (φ') is derived from the normalized cone resistance using the Robertson and Campanella correlation [25], and Young's modulus (E) is estimated as $E = 2.5q_c$ following Schmertmann [26] for square footings on sand. The effective overburden stress is computed assuming a unit weight of 18 kN/m^3 and a groundwater table at 2.0 m depth. Transformation uncertainties of COV = 0.08 for φ' and COV = 0.30 for E are adopted [7], [27]. The second model (V2) assumes cohesive soil behavior. The undrained shear strength (s_u) is obtained from $s_u = (q_c - \sigma_v)/N_k$ with a cone factor $N_k = 15$ [11], and Young's modulus is estimated as $E = 5.0q_c$, reflecting the higher compressibility of clays at equivalent cone resistance [36]. The unit weight is taken as 17 kN/m^3 with a groundwater table at 1.5 m, and transformation uncertainties of COV = 0.12 for s_u and COV = 0.35 for E are adopted. For both models, the total uncertainty of each derived parameter is computed as the root-sum-square of the inherent variability and the transformation uncertainty, following the framework of Phoon and Kulhawy [2], so that both the natural heterogeneity of the soil and the imperfection of the empirical correlations are accounted for.

The root-sum-square combination is justified because the two uncertainty sources are physically distinct: the inherent variability reflects spatial heterogeneity that would persist even with perfect measurement, whereas the transformation uncertainty reflects the scatter of the calibration database around the best-fit correlation line. Treating them independently is a standard assumption in geotechnical reliability practice [7], although in reality the transformation scatter may itself vary with soil type and stress level. The resulting total COV values, which range from 0.12 for the friction angle to 0.80 for Young's modulus, span more than an order of magnitude and directly control the uncertainty propagated through the Monte Carlo simulation.

D. Foundation Performance Models

Four design scenarios represent typical shallow foundation configurations: a small square footing (B = 1.0 m, D_f = 0.5 m) under applied pressures of 150 and 200 kPa, a medium square footing (B = 1.5 m, D_f = 1.0 m) at 200 kPa, and a large square footing (B = 2.0 m, D_f = 1.5 m) at 200 kPa. An allowable settlement of 25 mm is adopted as the serviceability limit criterion, representing a commonly used limiting value for isolated shallow foundations in geotechnical practice [6].

For the granular soil model (V1), the ultimate bearing capacity is computed with the Vesic formulation [37], in

which the ultimate bearing capacity q_{ult} is the sum of surcharge and self-weight contributions with bearing capacity factors that are exponential functions of the effective friction angle, together with shape and depth factors:

$$q_{ult} = q\,N_q s_q d_q + 0.5\,\gamma B N_\gamma s_\gamma d_\gamma \quad (1)$$

where N_q and N_γ are bearing capacity factors that are exponential functions of φ′, and s_q, s_γ, d_q, d_γ are shape and depth factors [37].

Settlement is computed using the Schmertmann strain influence factor method [26], which integrates the product of a strain influence factor, layer thickness, and the reciprocal of Young's modulus over a depth of influence equal to twice the footing width:

$$S = C_1 C_2\,\Delta p \sum \left(\frac{I_z\,\Delta z}{E}\right) \quad (2)$$

where C_1 and C_2 are embedment and creep correction factors, Δp is the net foundation pressure, and I_z is the strain influence factor [26]. The strain influence factor follows a triangular distribution, rising from 0.1 at the footing base to a peak that depends on the ratio of the net applied pressure to the effective overburden stress at the depth of peak influence.

The Schmertmann method is particularly well suited to the present framework because it consumes the CPT cone resistance directly through the stiffness correlation, avoiding the need for laboratory-determined compressibility parameters. The method incorporates two empirical correction factors: the embedment factor C_1, which reduces settlement when the footing is buried below ground level, and the time factor C_2, which accounts for creep over the design life, taken here as 50 years. Both factors are deterministic in this study, but their associated uncertainty is implicitly absorbed into the settlement model error term, which is consistent with how these correction factors were calibrated in the original database [26].

For the cohesive soil model (V2), the ultimate bearing capacity is computed under undrained conditions (φ = 0) following Skempton [38], with q_{ult} expressed as the product of the undrained shear strength, the bearing capacity factor N_c = 5.14, and shape and depth correction factors, plus the overburden surcharge:

$$q_{ult} = s_u N_c s_c d_c + \gamma D_f \quad (3)$$

where N_c = 5.14, s_c and d_c are shape and depth factors for undrained conditions [38].

Settlement is computed as immediate undrained elastic settlement using the classical elasticity solution, with a Poisson's ratio of 0.5 for saturated clay and an influence factor of 0.88 for a rigid square footing.

Both bearing capacity and settlement models incorporate a lognormal model error term with a mean of 1.0 (unbiased prediction) and a coefficient of variation representing the predictive uncertainty of each model: COV = 0.12 for bearing capacity [39], [40] and COV = 0.40 (sand) and 0.45 (clay) for settlement [41].

E. Monte Carlo Simulation Framework

Five random variables are defined for each soil model: the strength parameter (φ′ for V1 and s_u for V2), the stiffness parameter (E for both models), the soil unit weight, and the bearing capacity and settlement model errors. Table I summarizes their distributions. Latin Hypercube Sampling [42] is employed for stratified sampling across the five-dimensional parameter space with a fixed random seed for reproducibility, dividing each dimension into N equiprobable intervals and sampling once per interval. A convergence study evaluated the coefficient of variation of the failure probability estimate (COV of P_f) from N = 1,000 to 100,000 simulations. The COV of P_f stabilized below 0.005 at N = 50,000 for the settlement limit state in all scenarios, and the bearing capacity P_f remained zero at all simulation sizes. A base simulation size of N = 100,000 is therefore adopted for all reported results.

The limit state functions are defined as the safety margin: $g_{ULS} = q_{ult} - q_{applied}$ for bearing capacity and $g_{SLS} = S_{allow} - S_{calculated}$ for settlement, where failure corresponds to $g \leq 0$. All computations are vectorized with NumPy, allowing the 100,000 realizations to be processed in a single function call; the entire simulation for one scenario completes in approximately 0.08 seconds on a standard laptop computer.

Latin Hypercube Sampling was selected over simple random sampling because it guarantees that each of the five random variables is represented across its full range in every simulation, stratifying the marginal distributions into N equally probable intervals. This stratification reduces the variance of the failure probability estimator for a given sample size, which is important when estimating small probabilities from the tails of the distribution. The convergence criterion adopted in this study, a coefficient of variation of the P_f estimate below 0.005, is stricter than the value of 0.1 that is sometimes suggested as adequate in the literature, providing additional confidence in the reported values.

The deterministic variables are the footing dimensions (B, L, D_f), the applied pressure, and the allowable settlement of 25 mm.

TABLE I. RANDOM VARIABLES FOR THE MONTE CARLO SIMULATION

Variable	V1 Distribution	V1 Parameters	V2 Distribution	V2 Parameters
Strength	Normal	μ = 39.2°, σ = 3.3°	Lognormal	μ = 294 kPa, COV = 0.57
Stiffness	Lognormal	μ = 12.9 MPa, COV = 0.78	Lognormal	μ = 25.8 MPa, COV = 0.80
Unit weight	Normal	μ = 18 kN/m³, COV = 0.05	Normal	μ = 17 kN/m³, COV = 0.05
Model error (BC)	Lognormal	μ = 1.00, COV = 0.12	Lognormal	μ = 1.00, COV = 0.12
Model error (St)	Lognormal	μ = 1.00, COV = 0.40	Lognormal	μ = 1.00, COV = 0.45

F. Reliability and Sensitivity Metrics

The probability of failure is computed as the fraction of realizations with a non-positive safety margin, and the reliability index is obtained from the inverse standard normal cumulative distribution function:

$$P_f = \frac{1}{N}\sum_{i=1}^{N} I\left[g(X_i) \leq 0\right] \quad (4)$$

$$\beta = -\Phi^{-1}\left(P_f\right) \quad (5)$$

where N is the number of realizations, g is the limit state function, X_i is the i-th sampled parameter vector, and Φ^{-1} denotes the inverse standard normal cumulative distribution function.

A 95% confidence interval for P_f is computed with the Clopper-Pearson method for binomial proportions. Sensitivity analysis is performed at two levels. First, Spearman rank

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correlation coefficients are computed between each input random variable and the safety margin of both limit states; Spearman correlation captures monotonic associations without assuming linearity, which is appropriate for the non-linear relationship between soil parameters and foundation performance. Second, a conditional probability analysis, visualized as a tornado plot, splits each random variable at its sample median and computes the failure probability of the lower and upper halves separately, so that variables whose conditional P_f differs most from the baseline are identified as the dominant contributors to reliability.

III. Results

A. CPT Statistical Characterization

Table II presents the statistical summary of all 16 CPT soundings. The cone resistance exhibits substantial spatial variability, with per-CPT means ranging from 36.1 kg/cm^2 (S4) to 89.4 kg/cm^2 (S7) and COV values from 0.492 (S14) to 1.085 (S10), for a site-wide mean COV of 0.683. Fifteen of the 16 soundings fall in the high variability category (COV > 0.5). The vertical scale of fluctuation ranges from 0.37 m (S11) to 1.48 m (S13), with a site-wide mean of 0.80 m, indicating that significant property changes occur over vertical distances of less than one meter. Fig. 1 shows representative CPT profiles with their linear trends, and Fig. 2 compares the COV of all soundings against the low and high variability thresholds. The q_c distribution is approximately evenly split between normal and lognormal across the 16 soundings. From an engineering perspective, these statistics imply that designing on the basis of a single representative value of q_c, as is common in routine practice, would disregard a variability band of roughly one order of magnitude across the site. The short correlation distance further implies that the assumption of homogeneous layers in conventional design profiles is not supported by the data, and that any reliability assessment must explicitly propagate the measured variability into the performance calculations rather than masking it behind conservative averages.

TABLE II. Statistical Summary of the CPT Data (16 Soundings)

CPT	N	Depth (m)	$\bar{q}_c$ (kg/cm^2)	COV	θ_v (m)	Best-fit
S1	44	0.2–8.8	39.0	0.690	0.694	Normal
S2	42	0.2–8.4	56.0	0.635	0.542	Normal
S3	39	0.2–7.8	63.2	0.620	0.781	Normal
S4	37	0.2–7.6	36.1	0.851	1.272	Lognormal
S5	43	0.2–8.6	52.4	0.772	0.731	Lognormal
S6	28	0.2–6.0	73.5	0.727	0.537	Lognormal
S7	43	0.2–8.6	89.4	0.605	0.958	Normal
S8	34	0.2–6.8	69.5	0.527	1.002	Normal
S9	29	0.2–6.0	46.4	0.722	0.720	Lognormal
S10	15	0.2–3.0	47.7	1.085	0.494	Lognormal
S11	20	0.2–4.0	54.9	0.747	0.374	Normal
S12	27	0.2–5.4	61.0	0.756	0.770	Lognormal
S13	74	0.2–14.8	40.7	0.633	1.477	Lognormal
S14	70	0.2–14.0	41.1	0.492	0.712	Normal
S15	40	0.2–8.2	50.4	0.569	0.485	Lognormal
S16	40	0.2–8.0	45.9	0.583	1.268	Normal

Note: q_c is the cone resistance; COV is the coefficient of variation; θ_v is the vertical scale of fluctuation.

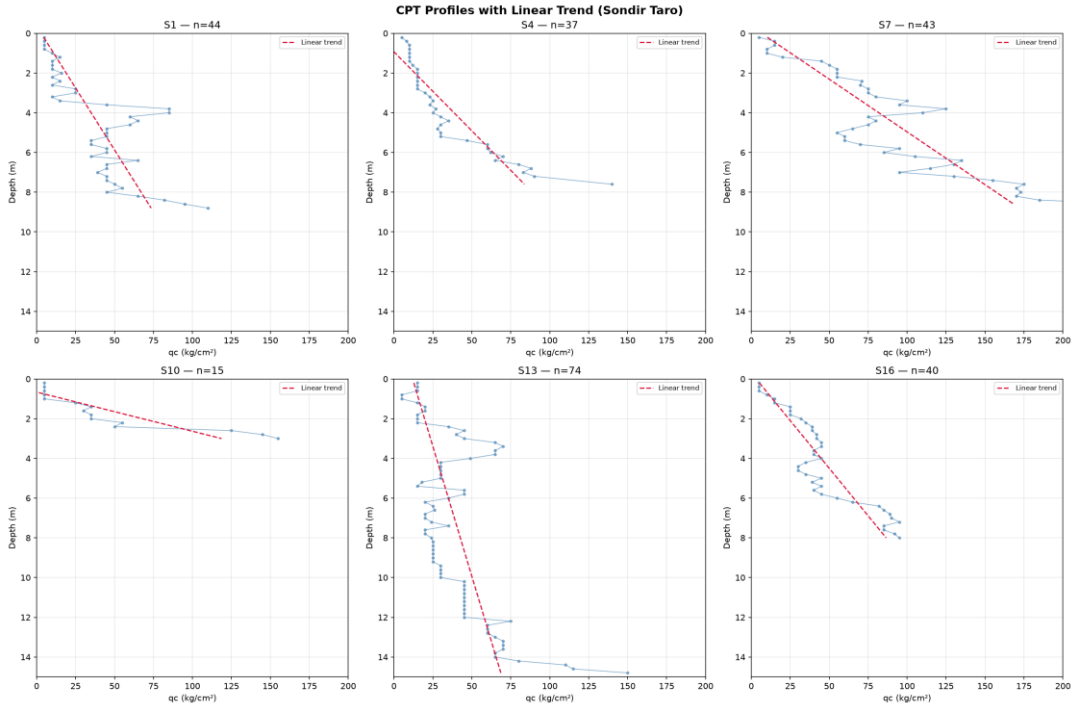

Fig. 1. CPT profiles with linear trends (six representative soundings).

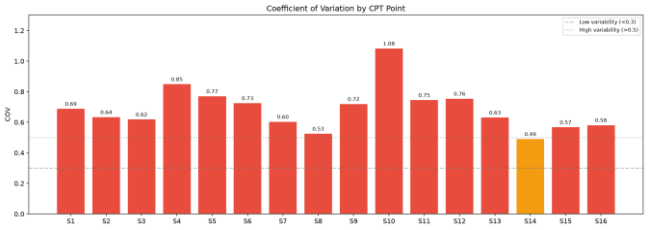

Fig. 2. Coefficient of variation of cone resistance by sounding.

B. *Derived Soil Parameters*

Table III summarizes the derived site-wide parameters. Under the sand assumption (V1), the mean effective friction angle is 39.2° with a COV of only 0.12, a low value that arises from the logarithmic compression inherent in the Robertson and Campanella correlation, and the mean Young's modulus is 12.9 MPa with a COV of 0.78. Under the clay assumption (V2), the mean undrained shear strength is 294 kPa with a COV of 0.57, and the mean Young's modulus is 25.8 MPa with a COV of 0.80, approximately twice the V1 value because the clay modulus factor ($\alpha = 5.0$) doubles the sand factor ($\alpha = 2.5$). The contrasting COV values between strength and stiffness parameters carry important implications: the low COV of φ' suggests modest variability in sand bearing capacity, whereas the high COV of E indicates that settlement will be highly variable regardless of the soil type assumption.

TABLE III. SITE-WIDE DERIVED SOIL PARAMETERS

Parameter	V1 — Sand	V2 — Clay
Strength	$\varphi' = 39.2 \pm 3.3°$ (COV = 0.12)	$s_u = 294 \pm 163$ kPa (COV = 0.57)
Stiffness	$E = 12.9 \pm 9.4$ MPa (COV = 0.78)	$E = 25.8 \pm 18.7$ MPa (COV = 0.80)
Strength distribution	Normal	Lognormal
Stiffness distribution	Lognormal	Lognormal

C. *Ultimate Limit State of Bearing Capacity*

For all four design scenarios and both soil models, the probability of bearing capacity failure is zero, with no failures observed in 100,000 Monte Carlo realizations per scenario, and the corresponding reliability indices exceed the numerical ceiling of 7.9 imposed when $P_f = 0$. The 5th-percentile factor of safety ranges from 3.4 to 9.5 under V1 and from 3.9 to 5.2 under V2 (Table IV), confirming that even the lower tail of the bearing capacity distribution lies well above the applied load. This result is a direct consequence of the relatively high soil strength at the site combined with moderate applied pressures. The zero failure probability does not imply perfect safety; rather, it reflects that 100,000 simulations cannot resolve failure probabilities below approximately 10^{-5}. Nevertheless, the ultimate limit state does not govern foundation design at this site.

TABLE IV. ULTIMATE LIMIT STATE RELIABILITY (BEARING CAPACITY, $P_F = 0$ FOR ALL SCENARIOS)

Scenario	V1 — Sand		V2 — Clay	
	$q_{ult,mean}$ (kPa)	FS (P_5)	$q_{ult,mean}$ (kPa)	FS (P_5)
Small, q = 150 kPa	1,809	4.5	2,187	5.2
Small, q = 200 kPa	1,809	3.4	2,187	3.9
Medium, q = 200 kPa	3,387	6.4	2,316	4.1
Large, q = 200 kPa	4,983	9.5	2,385	4.3

D. *Serviceability Limit State of Settlement*

Settlement reliability exhibits a dramatic dependence on the assumed soil model (Table V and Fig. 3). Under the sand assumption (V1), the mean settlement ranges from 39.6 mm (Small-150) to 84.8 mm (Large-200), exceeding the 25 mm allowable settlement in all scenarios. The probability of failure ranges from 0.573 to 0.875, with reliability indices between -1.15 and -0.18. Negative β values indicate that the expected settlement exceeds the allowable limit, a clearly unacceptable situation for engineering design. Under the clay assumption (V2), the mean settlement is substantially lower, from 5.9 mm to 14.7 mm, and the probability of failure ranges from 0.016 to 0.144, with reliability indices between +1.06 and +2.14. Although these β values remain below the target range of 2.9 to 4.2 recommended for foundations [39], they represent a dramatic improvement over the V1 results. Examining the percentile structure of the settlement distributions provides additional insight: under V2, the 5th percentile settlement is only 1 to 3 mm and the median is 4 to 10 mm, whereas the 95th percentile reaches 16 to 40 mm. The skewness of the distribution, inherited from the lognormal stiffness model, means that the majority of realizations are comfortable while a minority of soft-soil realizations drive the failure probability. This asymmetry is a fundamental characteristic of stiffness-driven serviceability problems and explains why deterministic design based on mean properties systematically underestimates the risk.

The 20 to 40 times the difference in P_f between V1 and V2 for identical foundation geometries and loads is one of the most striking findings of this study, and it arises primarily from the higher mean stiffness under the clay assumption (25.8 MPa versus 12.9 MPa), which shifts the entire settlement distribution downward, and from the different settlement calculation frameworks. An unexpected result is that P_f increases with footing width in both models, from 0.573 to 0.875 under V1 and from 0.016 to 0.144 under V2 as B increases from 1.0 to 2.0 m. This counter-intuitive trend occurs because the settlement influence zone deepens with increasing footing width, engaging a larger volume of compressible soil whose average stiffness does not increase proportionally.

TABLE V. SERVICEABILITY LIMIT STATE RELIABILITY (SETTLEMENT, S_{ALLOW} = 25 MM)

Scenario	V1 S_{mean} (mm)	V1 P_f	V1 β	V2 S_{mean} (mm)	V2 P_f	V2 β
Small-150	39.6	0.573	−0.18	5.9	0.016	+2.14
Small-200	56.8	0.740	−0.64	8.1	0.037	+1.79
Medium	72.5	0.830	−0.95	11.5	0.088	+1.35
Large	84.8	0.875	−1.15	14.7	0.144	+1.06

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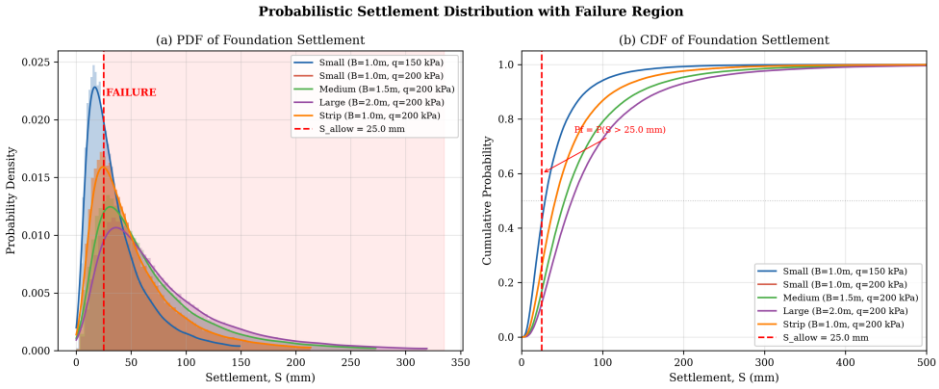


Fig. 3. Settlement distribution under the sand assumption (V1) with the failure region beyond the 25 mm allowable limit.

E. Sensitivity Analysis

Table VI and Fig. 4 present the sensitivity analysis for the representative Small-200 scenario. In both soil models, Young's modulus E emerges as the overwhelmingly dominant variable for settlement reliability, with Spearman correlation coefficients of +0.863 (V1) and +0.843 (V2), and the conditional probability analysis confirms this dominance: when E is below its median, the settlement P_f under V1 rises from the baseline of 0.740 to 0.981, whereas when E is above its median, P_f drops to 0.498, a swing of 0.483. The settlement model error is the second most influential variable, with Spearman correlations of -0.467 (V1) and -0.449 (V2) and a conditional P_f swing of 0.261 under V1. The bearing capacity model error and the unit weight exert negligible influence.

Critically, neither the friction angle (V1) nor the undrained shear strength (V2) shows any meaningful correlation with the settlement safety margin, despite governing the bearing capacity. Characterizing soil strength, which has traditionally been the primary focus of geotechnical site investigation, therefore provides essentially no information about the probability of settlement failure. It is the stiffness, and specifically the CPT-to-stiffness correlation, that controls whether a foundation design meets serviceability requirements.

TABLE VI. SENSITIVITY ANALYSIS FOR THE SMALL FOOTING (B = 1.0 M, Q = 200 KPA)

Variable	V1 ρ (SLS)	V1 Δ P_f	V2 ρ (SLS)	V2 Δ P_f
E (MPa)	+0.863	−0.483	+0.843	−0.074
ε_{st} (model St)	−0.467	+0.261	−0.449	+0.060
γ (kN/m^3)	+0.016	−0.012	−0.001	0.000
φ' / s_u	−0.002	+0.002	−0.014	−0.001
ε_{bc} (model BC)	+0.000	+0.001	+0.007	−0.001

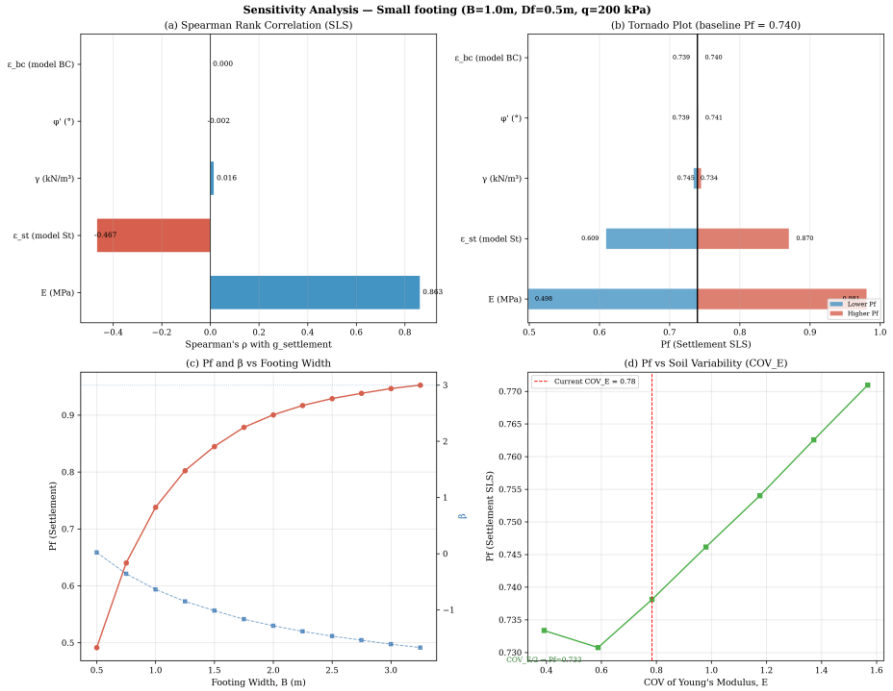
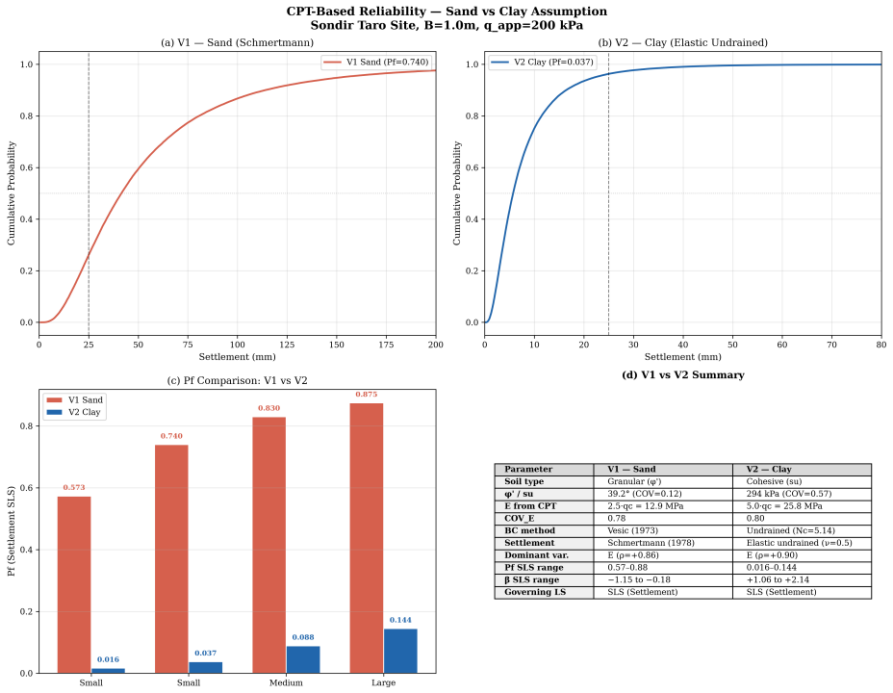


Fig. 4. Sensitivity analysis under the sand assumption (V1): Spearman correlation, tornado plot, and parametric trends.

F. Comparison of Soil Model Frameworks

Fig. 5 compares the two frameworks in terms of settlement CDFs, failure probabilities, and input parameters, and Fig. 6 presents the V2 sensitivity analysis. The systematic difference between V1 and V2 highlights a fundamental epistemological challenge in CPT-based reliability assessment: the engineer must decide which soil constitutive framework to apply before the reliability analysis can be performed, yet that decision, often based on incomplete subsurface information, determines whether the resulting P_f is interpreted as unacceptable (sand) or marginally acceptable (clay). CPT-based soil behavior type

classification [43] is therefore a prerequisite for meaningful reliability assessment.



Parameter	V1 — Sand	V2 — Clay
Soil type	Granular (φ′)	Cohesive (su)
φ′ / su	39.2° (COV=0.12)	294 kPa (COV=0.57)
E from CPT	2.5·qc = 12.9 MPa	5.0·qc = 25.8 MPa
COV_E	0.78	0.80
BC method	Vesic (1973)	Undrained (Nc=5.14)
Settlement	Schmertmann (1978)	Elastic undrained (ν=0.5)
Dominant var.	E (ρ=+0.86)	E (ρ=+0.90)
Pf SLS range	0.57–0.88	0.016–0.144
β SLS range	−1.15 to −0.18	+1.06 to +2.14
Governing LS	SLS (Settlement)	SLS (Settlement)

Fig. 5. Comparison of the sand (V1) and clay (V2) frameworks: settlement CDFs, failure probabilities, and summary of key parameters.

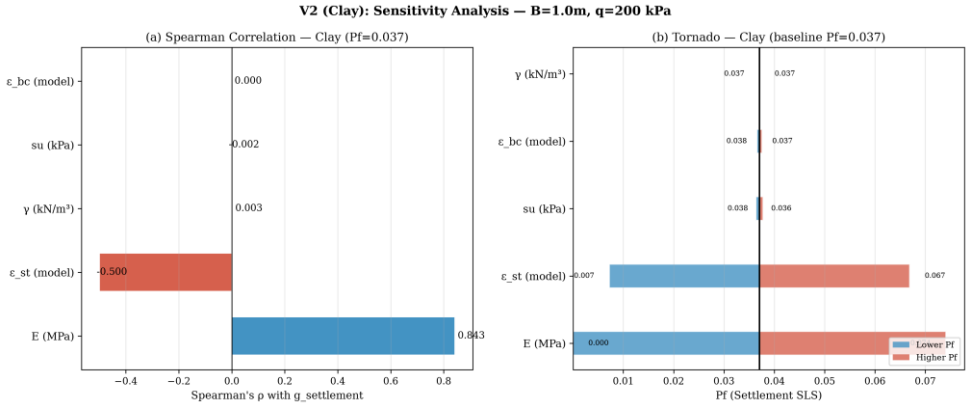


Fig. 6. Sensitivity analysis under the clay assumption (V2): Spearman correlation and tornado plot.

IV. Discussion

A. *The Governing Role of Settlement*

The finding that settlement, rather than bearing capacity, governs foundation reliability warrants careful interpretation. At this site, the deterministic factors of safety against bearing capacity failure range from 9 to 25 under the sand assumption and from 11 to 15 under the clay assumption, far exceeding the traditional FS of 3, because the soil is relatively strong ($\varphi' \approx 39°$ or $s_u \approx 294$ kPa) compared with the moderate applied pressures. The entire distribution of ultimate capacity therefore lies far above the applied load, even after accounting for strength variability. Settlement, by contrast, is governed by stiffness, and the stiffness parameters derived from CPT carry substantially higher variability: the COV of E, approximately 0.78 to 0.80, is nearly seven times the COV of φ' and 1.4 times the COV of s_u. This high stiffness variability, combined with the inverse relationship between settlement and E, produces a wide, positively skewed settlement distribution whose upper tail readily exceeds the allowable limit, particularly under the Schmertmann method, which amplifies settlement through its correction factors. This dominance of stiffness variability over strength variability reproduces the pattern reported in probabilistic settlement analyses of footings characterized by CPT data [44], [45].

B. *The Critical Role of Soil Model Selection*

The substantial disparity in settlement failure probability (P_f) between V1 and V2, with differences reaching 20–40 times, has direct implications for engineering decision-making. Under the sand assumption, a designer would conclude that the foundation configuration is fundamentally inadequate, requiring a larger footing, deeper embedment, ground improvement, or a deep foundation system. Under the clay assumption, the same designer might conclude that the design is marginally acceptable, particularly for a temporary structure or one with less stringent serviceability requirements. These are qualitatively different engineering outcomes from the same CPT data. The root cause lies in two interconnected decisions embedded in the soil model choice: the CPT-to-stiffness correlation factor differs by a factor of two between sand (2.5) and clay (5.0), directly halving the mean settlement

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under the clay assumption, and the settlement calculation framework changes fundamentally from the empirically calibrated Schmertmann method for sand to the theoretically derived elastic solution for clay. These differences are irreducible because they reflect genuine physical differences in how granular and cohesive soils respond to foundation loading, and no amount of additional site investigation can resolve which framework is correct without determining the actual soil type. For research, this establishes that CPT-based reliability studies must either incorporate soil behavior type classification explicitly — using the updated soil behaviour type framework [46], or a probabilistic classification scheme that carries the ambiguity through the analysis [18] — or acknowledge this sensitivity as a source of epistemic uncertainty. For practice, it reinforces the value of site investigation techniques that can distinguish drained from undrained soil behavior before reliability analysis is undertaken.

It is also worth noting that the two constitutive models represent endpoints of a spectrum; a transitional soil with intermediate drainage behavior would produce failure probabilities somewhere between the V1 and V2 bounds. The bracketing approach adopted here is therefore conservative in the sense that it quantifies the full range of plausible reliability outcomes, and the width of this range itself constitutes a measure of the epistemic uncertainty associated with soil classification. Reporting a single failure probability without acknowledging this bandwidth would give a false impression of precision, and this is a point that reliability-based design codes have not yet addressed explicitly.

C. The Dominance of Stiffness over Strength

The sensitivity analysis reveals a consistent pattern: regardless of the soil model, the strength parameters that have traditionally been the focus of geotechnical site investigation exert essentially zero influence on settlement reliability, whereas the stiffness parameter E, often treated as a secondary derived quantity, is the overwhelmingly dominant variable. This finding challenges a deeply ingrained practice. Site investigation programs are typically designed to characterize soil strength, while stiffness parameters are often estimated through empirical correlations whose uncertainty is rarely propagated into design calculations. For foundations where settlement is the governing limit state, which is commonly the case for competent soils under moderate loads, this emphasis is misplaced from a reliability perspective. Improving stiffness characterization, through seismic CPT, geophysical methods, or refined CPT-to-stiffness correlations, should yield greater improvements in foundation reliability than additional strength testing. Independent assessments of bearing capacity factor models reach a compatible conclusion, reporting modest model-to-model scatter relative to the variability of the underlying soil parameters [47], whereas uncertainty-aware capacity models trained on large databases still retain substantial predictive uncertainty [34]. However, the parametric study of P_f versus the COV of E reveals that halving the transformation uncertainty reduces P_f only modestly, from 0.740 to approximately 0.720 under V1, because the inherent variability of q_c, not the correlation uncertainty, dominates the total COV of E. At sites with inherently high spatial variability, such as the tropical volcanic deposit studied here, even perfect knowledge of the CPT-to-stiffness correlation would not fundamentally alter the reliability conclusion.

D. Engineering Implications

The traditional factor of safety of 3 for shallow foundations proves to be an inadequate reliability metric when settlement governs. For the Small-200 scenario under V1, the deterministic bearing capacity factor of safety is 9.0, yet the probability of exceeding the allowable settlement is 74%. Designing to a bearing capacity factor of safety, as is common practice, provides no assurance of adequate serviceability performance, and a reliability-based approach that explicitly considers settlement is necessary. Probabilistic analyses of square footings under comparable loads on CPT-characterized profiles reach the same conclusion [48].

The short vertical scales of fluctuation measured at the site (0.37 to 1.48 m) imply that soil properties decorrelate over distances of less than 1.5 m vertically, so closely spaced CPT measurements are necessary to capture the full spatial variability. The horizontal scale of fluctuation, which governs spatial averaging between CPT locations, could not be estimated from the available data because the 16 soundings were not arranged in a pattern that permits two-dimensional geostatistical analysis; future programs should consider a grid layout that enables estimation of both correlation structures [8]; the number and spacing of soundings required to resolve the governing correlation length can be established quantitatively before mobilisation [9], [49], and where budgets are constrained, sampling locations can be optimized against the expected reliability improvement rather than selected by judgement [35].

The comparison with code-based target reliability indices reveals a significant shortfall. Under V1, the achieved β values are negative for all scenarios, and under V2 they range from 1.06 to 2.14, both far below the target range of 2.9 to 4.2. The partial factors of Eurocode 7, calibrated using generic COV values substantially lower than the site-specific values observed here, may therefore not provide adequate reliability at sites with high inherent variability. A formal calibration of partial factors that accounts for site-specific COV, or the application of a higher global factor of safety where CPT data indicate a COV of q_c exceeding 0.5, would bring code-based design into closer alignment with reliability targets. Reliability-based design formulations that operate directly on site-specific statistics and transformed limit state functions offer a practical route to such a calibration [10].

A further implication concerns the interaction between variability and design life. The reliability indices reported here correspond to a 50-year design life through the creep correction factor of the settlement model, and they would degrade further for longer design lives. When combined with the observation that the target reliability indices in codes are defined for structural failure consequences, the serviceability shortfall identified in this study suggests that either the target indices for geotechnical serviceability should be revised, or the partial factors should be calibrated separately for settlement-controlled designs. The framework presented here provides the machinery for such a calibration, since it yields the full distribution of settlement from which any desired fractile-based design value can be extracted.

Finally, the finding that P_f increases with footing width challenges the conventional intuition that larger footings are inherently safer. Because increasing the footing width increases the total load at constant applied pressure and deepens the settlement influence zone, settlement becomes more severe and more uncertain with increasing width unless the stiffness increases with depth at a compensating rate. Designers should verify that increasing footing dimensions does not inadvertently degrade serviceability reliability.

E. Limitations

Several limitations should be acknowledged. The soil is treated as a homogeneous half-space with depth-averaged properties, neglecting the layering evident in the CPT profiles; a more refined analysis would classify each depth increment by soil behavior type and apply the appropriate correlation layer by layer. The settlement models do not account for time-dependent consolidation, which is significant for cohesive soils, so the elastic undrained settlement computed under V2 represents immediate settlement only. The random variables are treated as mutually independent, whereas φ' (or s_u) and E are both derived from q_c and exhibit positive correlation; accounting for this correlation would likely reduce the estimated P_f, because high q_c values simultaneously increase both strength and stiffness. The vertical scale of fluctuation is not explicitly incorporated into the simulation, and a full random field approach that generates correlated q_c profiles with the estimated θ_v would provide a more rigorous treatment of spatial averaging effects; stress-weighted spatial averaging formulations [19] and assessments of discretization error in random field realizations [23] provide the tools for that extension. Finally, the results are site-specific and should not be generalized without confirmation at other sites with different geological settings and variability characteristics.

V. Conclusion

This study developed and applied a Monte Carlo simulation framework for the reliability-based assessment of shallow foundation performance using CPT-derived spatial variability. The framework propagates uncertainty from raw CPT measurements through to the probability of failure without requiring finite element analysis, making reliability assessment accessible on standard computing hardware. Application to 16 CPTs at a tropical volcanic soil site with high inherent variability yields the following conclusions.

Settlement, not bearing capacity, governs foundation reliability at this site for all scenarios and both soil constitutive assumptions. The probability of bearing capacity failure is zero in 100,000 simulations per scenario, reflecting the high deterministic factors of safety achieved by the site's competent soils, whereas the probability of exceeding the 25 mm allowable settlement ranges from 0.016 to 0.875 depending on the scenario and soil model.

The assumed soil constitutive framework fundamentally alters reliability conclusions. Under the sand assumption, the settlement failure probability is 57% to 88% with negative reliability indices, whereas under the clay assumption it drops to 1.6% to 14.4%, a 20 to 40 fold reduction. CPT-based soil behavior type classification is therefore a critical prerequisite for meaningful reliability assessment.

Young's modulus is the overwhelmingly dominant uncertainty source for settlement reliability in both soil models, with Spearman rank correlation coefficients exceeding 0.84, whereas the strength parameters exert essentially zero influence on serviceability reliability. This finding challenges the traditional emphasis on strength characterization and suggests that improving stiffness characterization should be prioritized when settlement governs.

Reducing the transformation uncertainty of Young's modulus provides only modest reliability improvements because the inherent variability of cone resistance dominates the total stiffness uncertainty. At sites with inherently high spatial variability, achieving target reliability may require design modifications rather than additional site investigation alone, and the traditional factor of safety approach, which focuses exclusively on bearing capacity, is inadequate as a sole design criterion when settlement governs.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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