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On the Construction of Multipolar Intuitionistic Fuzzy Positive Implicative Ideal in BCK-Aljabar

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Abstract

This research focuses on the generalization of the concept of multipolar fuzzy intuitionistic ideal in BCK Algebra and bi-polar fuzzy intuitionistic positive implicative ideal in BCK algebra into multipolar fuzzy intuitionistic positive implicative ideal in BCK algebra. In this research, we define the concept of multipolar fuzzy intuitionistic positive implicative ideal in BCK algebra and give the examples. In addition, we examine the conditions that must be met by a multipolar fuzzy intuitionistic ideal in BCK algebra to become a multipolar fuzzy intuitionistic positive implicative ideal in BCK algebra. One of the theorems produced in this research is that a multipolar intuitionistic fuzzy $(\hat{P}, \hat{Q})$ of X is a multipolar intuitionistic fuzzy implicative positive ideal if only if $(\hat{P}, \hat{Q})$ is a multipolar intuitionistic fuzzy ideal of X and $\hat{P}(x * y) \geq \hat{P}((x * y) * y)$ and $\hat{Q}(x * y) \leq \hat{Q}((x * y) * y)$ apply, for all $x, y, z \in X$, where X is a BCK algebra.

Keywords: *BCK- Algebra, Fuzzy Set, Intuitionistic Fuzzy Multipolar, Intuitionistic Fuzzy Multipolar Ideal, Intuitionistic Fuzzy Multipolar Positive Implicative Ideal*

Introduction

BCK-algebras, introduced by Imai and Iseki in 1966, constitute one of the non-classical algebraic structures that emerged from the study of implication logic and deduction theory [1]. This structure has become an important object of investigation due to its ability to represent implicational relationships algebraically and its close connections with various branches of mathematics, including order theory, mathematical logic, and inference systems. Over time, several structural concepts have been developed to examine the internal properties of BCK-algebras, including sub-algebras [2], [3], congruence [4], and ideals [5]–[7]. Among these concepts, ideals play a central role because they can be used to characterize specific classes of BCK-algebras. One important generalization of the ideal concept is the notion of positive implicative ideals [8]–[11]. This concept was introduced to accommodate positive implication properties arising in various classes of BCK-algebras. Numerous studies have demonstrated that positive implicative ideals are closely related to commutative ideals [12]–[14], implicative ideals [15], and bounded BCK-algebras [16], [17]. Consequently, the development of positive implicative ideals not only enriches the theory of BCK-algebras but also provides a foundation for the emergence of more sophisticated generalizations of ideals.

On the other hand, the representation of uncertainty has become a significant issue in the development of modern mathematical models. To overcome the limitations of classical set theory in

handling uncertain information, Zadeh introduced fuzzy set theory, which allows each element to possess a degree of membership within the interval [0,1] [18]. Within algebraic structures, fuzzy concepts have been incorporated into various systems, including BCK-algebras, leading to the development of notions such as fuzzy BCK-sub algebras [19], fuzzy BCK-ideals [20], fuzzy implicative BCK-ideals [21], and fuzzy positive implicative BCK-ideals [22]. These studies have shown that the fuzzy approach provides a more flexible characterization than the classical approach. Nevertheless, representations based solely on a single membership function are not sufficient to simultaneously describe the degrees of acceptance and rejection of an object. To address this limitation, Atanassov introduced intuitionistic fuzzy sets, in which each element is characterized by both membership and non-membership functions satisfying certain conditions [23]. Since their introduction, intuitionistic fuzzy sets have been widely applied to various algebraic structures, including BCK-algebras, resulting in concepts such as intuitionistic fuzzy BCK-algebras [24], intuitionistic fuzzy BCK-ideals [25]–[27], and intuitionistic fuzzy positive implicative BCK-ideals [28].

Furthermore, for problems involving dual-sided information, such as satisfaction and dissatisfaction, profit and loss, or support and opposition, Zhang introduced the concept of bipolar fuzzy sets [29]. In bipolar fuzzy sets, each element is evaluated based on two mutually opposite types of information, enabling the simultaneous modeling of positive and negative aspects. This approach provides a more representative framework than intuitionistic fuzzy sets in decision-making contexts that require bidirectional evaluation [30]. Consequently, many researchers have applied bipolar fuzzy and bipolar intuitionistic fuzzy concepts to BCK-algebras [31]–[38]. As data complexity and the demand for multicriteria decision-making continue to increase, Chen et al. introduced multipolar fuzzy sets as a generalization of bipolar fuzzy sets [39]. In a multipolar fuzzy set, the membership degree of an element is represented by a vector in $[0,1]^m$, where each component corresponds to a distinct aspect or evaluation criterion. For example, the assessment of an educational institution cannot be reduced merely to good or bad; rather, it involves multiple dimensions such as academic quality, facilities, management effectiveness, and graduate competitiveness. Each of these dimensions can be expressed by an individual membership degree in [0,1], allowing the institution's overall quality to be represented as a multipolar vector.

Literature Review

In this section, we discuss the theories that are the basis for the development of the concept in the next chapter.

Definition 2.1: Suppose $(X,*,0)$ it is an empty set that contains special elements "0" and " $*$ " is a binary operation in the X so is called BCK-algebra if it meets the following axioms:

(i) $((x*y)*(x*z))*(z*y)=0$,

(ii) $(x*(x*y))*y=0$,

(iii) $x*x=0$,

(iv) $x*y=0$ and $y*x=0$, then $x=y$

(v) $0*x=0$,

for all $x, y, z \in X$.

Proposition 2.1: If $(X,*,0)$ is a BCK-algebra. Defines binary operations " $\leq$ " on X with assuming $x \leq y$ if and only if $x * y = 0$. Then the following properties occur:

(i) $x * 0 = x$,

(ii) $x * y \leq x$,

(iii) $(x * y) * z = (x * z) * y$,

(vi) $(x * z) * (y * z) \leq x * y$,

(iv) $x * \big(x * (x * y)\big) = x * y$,

(v) $x * y \leq z \Longrightarrow x * z \leq y$,

(vi) $\big((x * z) * z\big) * (y * z) \leq (x * y) * z$,

(vii) $(x * z) * \big(x * (x * z)\big) = (x * z) * z$,

(viii) $\Big(x * \big(y * (y * x)\big)\Big) * \Big(y * \Big(x * \big(y * (y * x)\big)\Big)\Big) \leq x * y$.

for all $x, y, z \in X$.

Definition 2.2: Suppose $(X,*,0)$ is a BC-algebra and an empty set are called $I \subseteq X$ is called the ideal of the BCK-algebra of X if it satisfies:

(i) $0 \in I$,

(ii) If $x * y \in I$ and $y \in I$, then $x \in I$,

for all $x, y \in X$.

Definition 2.3: Suppose $(X,*,0)$ that a BCK-algebra and an empty set are called $I \subseteq X$ implicative positive ideals of BCK-algebra of X if it meets:

(i) $0 \in I$,

(ii) If $(x * y) * z \in I$ and $(y * z) \in I$, then $(x * z) \in I$,

for all $x, y, z \in X$.

Definition 2.4: An m-polar fuzzy set $\hat{P}$ of X is called an m-polar fuzzy sub-algebra of X if it meets the following conditions:

$$(\forall x, y \in X)\ \big(\hat{P}(x * y) \geq \inf\{\hat{P}(x), \hat{P}(y)\}\big), \qquad (1)$$

in such that

$$(\forall x, y \in X)\ \Big(\big(\pi_i \circ \hat{P}\big)(x * y) \geq \inf\{\big(\pi_i \circ \hat{P}\big)(x), \big(\pi_i \circ \hat{P}\big)(y)\}\Big), \qquad (2)$$

for each $i = 1,2,3, \ldots m$.

Definition 2.5: Suppose $(X, 0,*)$ is a BCK-algebra. A multipolar intuitionistic fuzzy of X is a mapping

$$\big(\hat{P}, \hat{Q}\big): X \to ([0,1] \times [0,1])^m \qquad (3)$$

$$x \mapsto \big(\hat{P}(x), \hat{Q}(x)\big)$$

where $\hat{P}: X \to [0,1]^m$ and $\hat{Q}: X \to [0,1]^m$ is the multipolar fuzzy set of X such that

$$\hat{P}(x) + \hat{Q}(x) \leq 1, \quad (4)$$

then apply

$$\left((\pi_i \circ \hat{P})(x) + (\pi_i \circ \hat{Q})(x) \leq 1\right), \quad (5)$$

for each $i = 1,2,3, \ldots m$.

Definition 2.6: Suppose $(X, 0, *)$ is a BCK-algebra. A multipolar intuitionistic fuzzy $(\hat{P}, \hat{Q})$ of X is called a multipolar intuitionistic fuzzy ideal of X if it satisfies:

(i) $(\forall x, y \in X)\ \left(\hat{P}(0) \geq \hat{P}(x) \text{ and } \hat{Q}(0) \leq \hat{Q}(x)\right)$,

in such that

$$\left((\pi_i \circ \hat{P})(0) \geq (\pi_i \circ \hat{P})(x) \text{ and } (\pi_i \circ \hat{Q})(0) \leq (\pi_i \circ \hat{Q})(x)\right),$$

(ii) $(\forall x, y \in X)\ \left(\hat{P}(x * y) \geq \inf\{\hat{P}(x), \hat{P}(y)\} \text{and } \hat{Q}(x * y) \leq \sup\{\hat{Q}(x), \hat{Q}(y)\}\right)$,

in such that

$(\pi_i \circ \hat{P})(x) \geq \inf\{(\pi_i \circ \hat{P})(x * y), (\pi_i \circ \hat{P})(y)\}$ and

$(\pi_i \circ \hat{Q})(x) \leq \sup\{(\pi_i \circ \hat{Q})(x * y), (\pi_i \circ \hat{Q})(y)\}$,

for each $i = 1,2,3, \ldots m$.

Results

In this section, we will discuss we define the concept of multipolar fuzzy intuitionistic positive implicative ideal in BCK algebra and give the examples. In addition, we examine the conditions that must be met by a multipolar fuzzy intuitionistic ideal in BCK algebra to become a multipolar fuzzy intuitionistic positive implicative ideal in BCK algebra.

Definition 3.1: Suppose $(X, 0, *)$ it is a BCK-algebra. A multipolar intuitionistic fuzzy $(\hat{P}, \hat{Q})$ of X is called a multipolar intuitionistic fuzzy implicative positive ideal of X if it meets:

(i) $(\forall\, x \in X)\ \left(\hat{P}(0) \geq \hat{P}(x) \text{ and } \hat{Q}(0) \leq \hat{Q}(x)\right)$,

in such that

$$\left((\pi_i \circ \hat{P})(0) \geq (\pi_i \circ \hat{P})(x) \text{ and } (\pi_i \circ \hat{Q})(0) \leq (\pi_i \circ \hat{Q})(x)\right),$$

(ii) $(\forall x, y, z \in X)\ \left(\hat{P}(x * z) \geq \inf\{\hat{P}((x * y) * z), \hat{P}(y * z)\} \text{and } \hat{Q}(x * z) \leq \sup\{\hat{Q}((x * y) * z), \hat{Q}(y * z)\}\right)$,

in such that

$(\pi_i \circ \hat{P})(x * z) \geq \inf\{(\pi_i \circ \hat{P})((x * y) * z), (\pi_i \circ \hat{P})(y * z)\}$ and

$(\pi_i \circ \hat{Q})(x * z) \leq \sup\{(\pi_i \circ \hat{Q})((x * y) * z), (\pi_i \circ \hat{Q})(y * z)\}$,

for each $i = 1,2,3, \ldots m$.

Example 3.1: Suppose $X = \{0, a, b, c, d\}$ is a set with binary operations " $*$ " as shown in Table 1:

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Table 1. Cayley Table for $(X,*,0)$

$*$	$\mathbf{0}$	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$
0	0	0	0	0	0
a	a	0	a	a	a
b	b	b	0	b	b
c	c	c	c	0	c
d	d	d	d	d	0

Thus, $(X,*,0)$ is a BCK-algebra. Suppose a multipolar intuitionistic fuzzy $(\hat{P},\hat{Q})$ of X is defined as follows:

$$(\hat{P},\hat{Q}): X \to [0,1]^4 \times [0,1]^4$$

$$\text{For any } \omega \mapsto \begin{cases} \big((0.75,0.31),(0.69,0.42),(0.82,0.28),(0.71,0.37)\big), & if\ \omega = 0, \\ \big((0.65,0.42),(0.59,0.60),(0.82,0.49),(0.71,0.43)\big), & if\ \omega = a, \\ \big((0.44,0.47),(0.49,0.51),(0.58,0.33),(0.52,0.37)\big), & if\ \omega = b, \\ \big((0.40,0.48),(0.29,0.58),(0.43,0.56),(0.42,0.37)\big), & if\ \omega = c, \\ \big((0.41,0.51),(0.39,0.47),(0.47,0.53),(0.62,0.51)\big), & if\ \omega = d. \end{cases}$$

Since conditions (i) and (ii) of Definition 3.1 are met, it is evident that $(\hat{P},\hat{Q})$ is a multipolar intuitionistic fuzzy implicative positive ideal.

Theorem 3.1: A multipolar intuitionistic fuzzy implicative positive ideal $(\hat{P},\hat{Q})$ of X is a multipolar intuitionistic fuzzy ideal of X but not the other way around.

Proof: Suppose $(\hat{P},\hat{Q})$ is a multipolar intuitionistic fuzzy implicative positive ideal of X, then based on Definition 3.1 applies

(i) $(\forall\, x \in X)\ \big(\hat{P}(0) \geq \hat{P}(x) \text{ and } \hat{Q}(0) \leq \hat{Q}(x)\big)$,

in such that

$$\big((\pi_i \circ \hat{P})(0) \geq (\pi_i \circ \hat{P})(x) \text{ and } (\pi_i \circ \hat{Q})(0) \leq (\pi_i \circ \hat{Q})(x)\big),$$

(ii) $(\forall x,y,z \in X)\ \big(\hat{P}(x*z) \geq \inf\{\hat{P}((x*y)*z), \hat{P}(y*z)\} \text{ dan } \hat{Q}(x*z) \leq \sup\{\hat{Q}((x*y)*z), \hat{Q}(y*z)\}\big)$,

in such that

$(\pi_i \circ \hat{P})(x*z) \geq \inf\{(\pi_i \circ \hat{P})((x*y)*z), (\pi_i \circ \hat{P})(y*z)\}$ and

$(\pi_i \circ \hat{Q})(x*z) \leq \sup\{(\pi_i \circ \hat{Q})((x*y)*z), (\pi_i \circ \hat{Q})(y*z)\}$,

If the value $z = 0$, then part (ii) becomes,

$\big(\hat{P}(x) \geq \inf\{\hat{P}(x*y), \hat{P}(y)\} \text{ and } \hat{Q}(x) \leq \sup\{\hat{Q}(x*y), \hat{Q}(y)\}\big)$,

in such that

$(\pi_i \circ \hat{P})(x) \geq \inf\{(\pi_i \circ \hat{P})(x*y), (\pi_i \circ \hat{P})(y)\}$ and

$(\pi_i \circ \hat{Q})(x) \leq \sup\{(\pi_i \circ \hat{Q})(x*y), (\pi_i \circ \hat{Q})(y)\}$.

Thus, from (i) and (ii) it is evident that $(\hat{P},\hat{Q})$ it is a multipolar intuitionistic fuzzy ideal of X.

Here is an example for Theorem 3.1 statement.

Example 3.2: Suppose $X = \{0, a, b, c, d\}$ is a set with binary operations " $*$ "as shown in Table 2.

Table 2. Cayley Table for $(X,*,0)$

$*$	**0**	***a***	***b***	***c***
0	0	0	0	0
a	a	0	a	a
b	b	a	0	b
c	c	c	c	0

Thus $(X,*,0)$ is a BCK-algebra. Suppose a multipolar intuitionistic fuzzy $(\hat{P},\hat{Q})$ of $X$is defined as follows:

$$(\hat{P},\hat{Q}): X \to [0,1]^3 \times [0,1]^3$$

$$\omega \mapsto \begin{cases} ((0.6,0.3),(0.6,0.2),(0.09,0.01)), & if\ \omega = 0, \\ ((0.5,0.4),(0.5,0.4),(0.08,0.03)), & if\ \omega = a,b, \\ ((0.3,0.6),(0.3,0.7),(0.03,0.08)), & if\ \omega = c, \end{cases}$$

Since conditions (i) and (ii) of Definition 2.6 are met, it is evident that $(\hat{P},\hat{Q})$ *is a* multipolar intuitionistic fuzzy ideal, but not a multipolar intuitionistic fuzzy implicative positive ideal because

$$\hat{P}(b*a) < \inf\{\hat{P}((b*a)*a), \hat{P}(a*a)\}$$
$$\hat{P}(a) < \inf\{\hat{P}(a*a), \hat{P}(0)\}$$
$$\hat{P}(a) < \inf\{\hat{P}(0), \hat{P}(0)\}$$
$$\hat{P}(a) < \hat{P}(0)$$
$$(0.5,0.5,0.08) < (0.6,0.6,0.09),$$

and

$$\hat{Q}(b*a) > \text{sub}\{\hat{P}((b*a)*a), \hat{P}(a*a)\}$$
$$\hat{Q}(a) > \sup\{\hat{P}(a*a), \hat{P}(0)\}$$
$$\hat{Q}(a) > \sup\{\hat{P}(0), \hat{P}(0)\}$$
$$\hat{Q}(a) > \hat{P}(0)$$
$$(0.4,0.4,0.03) > (0.3,0.2,0.01).$$

Furthermore, the conditions for a multipolar intuitionistic fuzzy ideal become a multipolar intuitionistic fuzzy implicative positive ideal.

Theorem 3.2: A multipolar intuitionistic fuzzy $(\hat{P},\hat{Q})$ of X is a multipolar intuitionistic fuzzy implicative positive ideal of X if and only if $(\hat{P},\hat{Q})$ is a multipolar intuitionistic fuzzy ideal of X and applies

$$\hat{P}(x*y) \geq \hat{P}((x*y)*y) \text{ and } \hat{Q}(x*y) \leq \hat{Q}((x*y)*y). \quad (6)$$

Proof:

Suppose $(\hat{P},\hat{Q})$ a multipolar intuitionistic fuzzy implicative positive ideal of X, then

1. based on Theorem 3.1, $(\hat{P},\hat{Q})$ is a multipolar intuitionistic fuzzy ideal of X.
2. based on Definition 3.1 part (ii) if z replaced by y obtained,

$$\hat{P}(x*z) \geq \inf\{\hat{P}((x*y)*z), \hat{P}(y*z)\}$$

$$\hat{P}(x*y) = \inf\{\hat{P}((x*y)*y), \hat{P}(y*y)\}$$
$$= \inf\{\hat{P}((x*y)*y), \hat{P}(0)\}$$
$$= \hat{P}((x*y)*y),$$

and

$$\hat{Q}(x*z) \leq \sup\{\hat{P}((x*y)*z), \hat{P}(y*z)\}$$
$$\hat{Q}(x*y) = \sup\{\hat{P}((x*y)*y), \hat{P}(y*y)\}$$
$$= \sup\{\hat{P}((x*y)*y), \hat{P}(0)\}$$
$$= \hat{P}((x*y)*y),$$

for all $x, y, z \in X$.

Conversely, suppose $(\hat{P}, \hat{Q})$ is a multipolar intuitionistic fuzzy ideal of, then X

1. based Definition 2.6 part (i) obtained,

$$\left(\hat{P}(0) \geq \hat{P}(x) \text{ and } \hat{Q}(0) \leq \hat{Q}(x)\right), \text{for all } x \in X.$$

2. based on the Proposition 2.1 part (vi) that

$$((x*z)*z)*(y*z) \leq (x*y)*z,$$

then obtained,

$$\hat{P}\left(((x*z)*z)*(y*z)\right) \geq \hat{P}((x*y)*z) \text{ and } \hat{Q}\left(((x*z)*z)*(y*z)\right) \leq \hat{Q}((x*y)*z).$$

Based on the assumption that,

$$\hat{P}(x*y) \geq \hat{P}((x*y)*y) \text{ and } \hat{Q}(x*y) \leq \hat{Q}((x*y)*y),$$

if the value of y is replaced by z, then obtained

$$\hat{P}(x*z) \geq \hat{P}((x*z)*z)$$
$$\geq \inf\left\{\hat{P}\left(((x*z)*z)*(y*z)\right), \hat{P}(y*z)\right\}$$
$$\geq \inf\{\hat{P}((x*y)*z), \hat{P}(y*z)\},$$

and

$$\hat{Q}(x*z) \leq \hat{Q}((x*z)*z)$$
$$\leq \sup\left\{\hat{P}\left(((x*z)*z)*(y*z)\right), \hat{P}(y*z)\right\}$$
$$\leq \sup\{\hat{P}((x*y)*z), \hat{P}(y*z)\}.$$

Thus, $(\hat{P}, \hat{Q})$ is a multipolar intuitionistic fuzzy implicative positive ideal of X.

Conclusion

In this article, a model of a multipolar intuitionistic fuzzy implicative positive ideal of a BCK-algebra is constructed and examples are given and explain the conditions for a multipolar intuitionistic fuzzy ideal of a BCK algebra to be a multipolar intuitionistic fuzzy implicative positive ideal of a BCK algebra. For further research, it is possible to float the definition the model of a multipolar intuitionistic fuzzy implicative positive ideal from a BCK-algebra on other ideal forms such as a multipolar intuitionistic fuzzy implicative positive ideal-(ϵ, ϵ) and a multipolar intuitionistic fuzzy implicative positive ideal -$(\bar{\epsilon}, \bar{\epsilon})$.

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